Optimal Disinflation Under Learning

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(Preliminary and Incomplete)

October 2009

1 Introduction

A newly appointed central bank governor inherits a high average inflation rate from the past and wants to disinflate.

The bank has no official inflation target and lacks the political authority unilaterally to set one.

However, it has some flexibility in choosing how to implement its vague mandate.

What is the optimal policy?

1.1 Sargent (1982) "The Ends of Four Big Inflations"

By reforming institutions and changing the rules of the game, the central bank might be able to persuade the private sector that a new low-inflation policy is its best response.

In that case, the central bank can engineer a sudden, sharp disinflation at low cost in terms of lost output.

We take this off the table by assuming no institutional change, just a change of personnel.

We also take away the assumption of rational expectations, instead assuming that the private sector must learn the new policy.

Our scenario is more like the Volcker disinflation.

1.2 Goodfriend and King (2005) “The Incredible Volcker Disinflation”

Because no changes were made in the rules of the game, the private sector was initially unconvinced that Volcker would disinflate.

In contrast to Sargent’s examples, the Volcker disinflation was very costly.

1.3 Erceg and Levin (2003) "Imperfect Credibility and Inflation Persistence"

Volcker's policy was not transparent, at least at the beginning.

Learning about the long-run inflation target makes inflation more persistent relative to what it would be under rational expectations.

That increases the sacrifice ratio and produces output losses like those seen in the early 1980s.

1.4 What We Do

The analysis of Erceg, Levin, Goodfriend, and King is positive.

We address a normative question: what is optimal when the private sector is not persuaded in advance and must learn about the shift in policy?

Are Sargent's recommendations still wise? Or should the disinflation be more gradual, e.g. as recommended by Gordon (1982)?

We study a dynamic new Keynesian model modified so that target inflation is nonzero (Ascari 2004 and Sbordone 2007)

The central bank commits to a Taylor type rule and chooses its coefficients by minimizing a discounted quadratic loss function.

The private sector learns about the policy coefficients via Bayesian updating.

The central bank takes learning into account when solving its decision problem.

1.5 Main Lessons

Some policies that work well under rational expectations generate catastrophic losses under learning.

But some good policies exist that generate rapid transitions with little loss.

The central bank can obtain results close to those of Sargent even when the public is skeptical that the central bank will change its policy.

Transparency of the new regime is critical, for that accelerates learning.

2 A New-Keynesian Model with Non-Zero Target Inflation

We study a standard new Keynesian model modified in two ways.

- We loglinearize around a non-zero level of target inflation.
- The private sector must learn about target inflation.

IS Curve

$$y_t - \overline{y}_{t-1} = E_t^* \left(y_{t+1} - \overline{y}_{t-1} - (i_t - \pi_{t+1} - r) + g_{t+1} - g \right) + \varepsilon_{yt}$$

y_t is (log) output and $\overline{y}_{t-1}$ is its steady-state

i_t is the nominal interest rate

g_t is the rate of growth of technology

$$g_t = \left(1 - \rho_g\right) g + \rho_g g_{t-1} + \varepsilon_{gt},$$

$\varepsilon_{gt}, \varepsilon_{yt}$ are white noise shocks.

E_t^* represents a subjective expectations operator.

Aggregate supply block

$$\begin{aligned}
\pi_t - \bar{\pi}_{t-1} &= \kappa_{t-1}(y_t - \overline{y}_{t-1}) + \beta_{t-1}E_t^*(\pi_{t+1} - \bar{\pi}_{t-1}) + \varsigma_{t-1}\left(\delta_t - \overline{\delta}_{t-1}\right) \\
&\quad + \gamma_{1t-1}E_t^*[(\theta-1)(\pi_{t+1} - \bar{\pi}_{t-1}) + \phi_{t+1}] + u_t + \varepsilon_{\pi t}, \\
\phi_t &= \gamma_{2t-1}E_t^*[(\theta-1)(\pi_{t+1} - \bar{\pi}_{t-1}) + \phi_{t+1}], \\
\delta_t - \overline{\delta}_{t-1} &= \lambda_{1t-1}(\pi_t - \bar{\pi}_{t-1}) + \lambda_{2t-1}\left(\delta_{t-1} - \overline{\delta}_{t-1}\right).
\end{aligned}$$

π_t is inflation

$\bar{\pi}_{t-1}$ is the period $t-1$ estimate of target inflation

$\delta_t = \log\left[\int (p_{it}/p_t)^{-\theta}\, di\right]$ measures the resource cost of price dispersion.

ϕ_t is a dummy variable handy for writing the model in first-order form

u_t is a cost-push shock that evolves as

$$u_t = \rho_u u_{t-1} + \varepsilon_{ut}$$

$\varepsilon_{ut}, \varepsilon_{\pi t}$ are white noise shocks

Parameters

$$
\begin{aligned}
\beta_t &= \beta(1+\bar{\pi}_t) \\
\kappa_t &= (1+\nu)\,[1-\alpha(1+\bar{\pi}_t)^{\theta-1}][1-\alpha\beta(1+\bar{\pi}_t)^{\theta}]/\alpha(1+\bar{\pi}_t)^{\theta-1} \\
\gamma_{1t} &= \beta\bar{\pi}_t[1-\alpha(1+\bar{\pi}_t)^{\theta-1}] \\
\gamma_{2t} &= \alpha\beta(1+\bar{\pi}_t)^{\theta-1} \\
\varsigma_t &= \nu[1-\alpha(1+\bar{\pi}_t)^{\theta-1}][1-\alpha\beta(1+\bar{\pi}_t)^{\theta}]/\alpha(1+\bar{\pi}_t)^{\theta-1} \\
\lambda_{1t} &= \alpha\theta\bar{\pi}_t(1+\bar{\pi}_t)^{\theta-1}/\left(1-\alpha(1+\bar{\pi}_t)^{\theta-1}\right) \\
\lambda_{2t} &= \alpha(1+\bar{\pi}_t)^{\theta}
\end{aligned}
$$

β is the subjective discount factor

$1-\alpha$ is the probability of resetting price

θ is the elasticity of substitution across good varieties

ν is the inverse of the Frish elasticity of labor supply

In addition, the steady states for real variables also depend on $\bar{\pi}_t$

Monetary policy

$$i_t - i_{t-1} = \psi_\pi(\pi_{t-1} - \bar{\pi}) + \psi_x(y_{t-1} - y_{t-2}) + \varepsilon_{it}$$

The timing assumption follows McCallum.

We chose this functional form because it works well in environments like ours

- Learning: Orphanides and Williams 2007
- Non-zero target inflation: Cobion and Gorodnichenko 2008)

Loss function

$$L = E_0 \sum_t \beta^t [(\pi_t - \bar{\pi})^2 + \lambda(y_t - \overline{y})^2]$$

3 Bayesian Learning within a DSGE Model

Timing protocol

Conjecture a perceived law of motion (PLM)

Derive the actual law of motion (ALM) under the PLM

Verify that the PLM is the perceived ALM

Use the ALM to derive the likelihood function

Combine with agents' prior and find the posterior mode

3.1 The Timing Protocol

Everyone knows the model of the economy and form of the policy rule, but private agents do not know the policy parameters, $\psi = [\bar{\pi}, \psi_\pi, \psi_x, \sigma_i]$

Private agents enter period t with beliefs inherited from $t-1$, ψ_{t-1}

They treat estimated parameters as if they were known with certainty and form expectations accordingly.

At the same time, the central bank sets the systematic part of its instrument rule.

After that, shocks are realized and current-period outcomes are determined.

After observing those outcomes, private agents update their estimates ψ_t and carry them forward to $t+1$.

3.2 The Perceived Law of Motion

The private sector's beliefs can be characterized as a system of expectational difference equations,

$$A_{t-1}S_t = B_{t-1}E_t^* S_{t+1} + C_{t-1}S_{t-1} + D_{t-1}\varepsilon_t. \tag{1}$$

$S_t = [\pi_t, \phi_t, \delta_t, u_t, y_t, g_t, y_{t-1}, i_t, 1]'$

$\varepsilon_t = [\varepsilon_{\pi t}, \varepsilon_{ut}, \varepsilon_{xt}, \varepsilon_{gt}, \varepsilon_{it}]'$

$A_{t-1}, B_{t-1}, C_{t-1}$, and D_{t-1} have time subscripts because they depend on estimated policy coefficients ψ_{t-1}.

Conjecture that the PLM is a $VAR(1)$ with time-varying parameters,

$$S_t = F_{t-1}S_{t-1} + G_{t-1}\varepsilon_t. \tag{2}$$

Under the timing protocol,

$$E_t^* S_{t+1} = F_{t-1}^2 S_{t-1}. \tag{3}$$

Substitute (3) into (1) and re-arrange terms,

$$S_t = A_{t-1}^{-1}[B_{t-1}F_{t-1}^2 + C_{t-1}]S_{t-1} + A_{t-1}^{-1}D_{t-1}\varepsilon_t. \tag{4}$$

This verifies the conjecture.

Solve for F_{t-1}, G_{t-1} by undetermined coefficients

$$B_{t-1}F_{t-1}^2 - A_{t-1}F_{t-1} + C_{t-1} = 0 \tag{5}$$

$$G_{t-1} = A_{t-1}^{-1}D_{t-1}$$

3.3 The ALM under the PLM

The ALM depends on both the actual and perceived policy coefficients

- Actual, because that is what governs central-bank behavior
- Perceived, because that is what guides private-sector behavior

The structural form for the ALM is

$$A_{t-1}S_t = B_{t-1}E_t^* S_{t+1} + C_{at-1}S_{t-1} + D_{t-1}\varepsilon_t, \tag{6}$$

where A_{t-1}, B_{t-1}, and D_{t-1} are the same as above and $C_{at-1} \neq C_{t-1}$ is a matrix involving the actual policy coefficients.

Use the PLM to find the subjective expectations,

$$E_t^* S_{t+1} = F_{t-1}^2 S_{t-1}.$$

Substitute into the structural ALM,

$$A_{t-1} S_t = (B_{t-1} F_{t-1}^2 + C_{at-1}) S_{t-1} + D_{t-1} \varepsilon_t. \tag{7}$$

Solve for the reduced-form ALM,

$$S_t = H_{t-1} S_{t-1} + J_{t-1} \varepsilon_t, \tag{8}$$

where

$$\begin{aligned} H_{t-1} &= A_{t-1}^{-1} (B_{t-1} F_{t-1}^2 + C_{at-1}), \\ J_{t-1} &= A_{t-1}^{-1} D_{t-1}. \end{aligned} \tag{9}$$

3.4 Verify that the PLM is the perceived ALM

The reduced forms for the ALM and PLM are both $VAR(1)$.

The reduced-form ALM matrices solve

$$
\begin{aligned}
H_{t-1} &= A_{t-1}^{-1}(B_{t-1}F_{t-1}^{2} + C_{at-1}), \\
J_{t-1} &= A_{t-1}^{-1}D_{t-1}.
\end{aligned}
$$

The reduced-form PLM matrices solve

$$
\begin{aligned}
F_{t-1} &= A_{t-1}^{-1}(B_{t-1}F_{t-1}^{2} + C_{t-1}), \\
G_{t-1} &= A_{t-1}^{-1}D_{t-1}.
\end{aligned}
$$

It follows that $G_{t-1} = J_{t-1}$.

By inspection, one can verify that C_{at-1} and C_{t-1} are identical expect that C_{at-1} depends on actual policy coefficients while C_{t-1} depends on perceived policy coefficients.

If we replace the actual coefficients in C_{at-1} with the perceived coefficients, we obtain C_{t-1}.

It follows that the perceived $H_{t-1} = F_{t-1}$.

Hence, the perceived ALM coincides with the PLM.

3.5 The Likelihood Function for the Private Sector's Approximating Model

The observables X_t are a subset of the state vector S_t,

$$X_t = e_X S_t,$$

where e_X is a selection matrix.

Using the prediction-error decomposition, the likelihood function is

$$p(X^t|\psi) = p(X_0|\psi) \prod_{j=1}^{t} p(X_j|X^{j-1}, \psi), \tag{10}$$

where ψ are the unknown policy coefficients.

The private sector's approximating model has the same form as the ALM.

According to the ALM, X_t is conditionally normal with mean

$$m_{t|t-1} = e_X H_{t-1} S_{t-1}, \tag{11}$$

and variance

$$V_{t|t-1} = e_X J_{t-1} var(\varepsilon_t) J'_{t-1} e'_X. \tag{12}$$

It follows that the log likelihood function is

$$\ln p(X^t|\psi) = -\frac{1}{2}\sum_{j=1}^{t} \ln |V_{j|j-1}| - \frac{1}{2}(X_j - m_{j|j-1})' V_{j|j-1}^{-1}(X_j - m_{j|j-1}). \tag{13}$$

3.6 The Private Sector's Prior

Below we posit a variety of priors $p(\psi)$.

3.7 The Private Sector's Posterior

By Bayes' theorem, the log of their posterior kernel is

$$\ln p(\psi|X^t) = \ln p(X^t|\psi) + \ln p(\psi)$$

Because of our anticipated-utility assumption, we only need a point estimate (the posterior mode) at each date.

4 Calibration

- From Cogley-Sbordone (AER 2008):

$$\alpha = 0.6,\ \beta = 0.997,\ \theta = 10$$

  Also, there is no indexation, in line with our estimates.

- Labor disutility parameters

$$\chi = 1,\ \nu = 0.5$$

- Abstract from growth, $g = 0$

- From Cogley, Sargent and Primiceri (2009), we take parameters for the shocks processes u_t and g_t,

$$\rho_u = 0.4,\ 100\sigma_u = 0.12,\ \rho_g = 0.27,\ 100\sigma_g = 0.5$$

- We arbitrarily set

$$\sigma_y = \sigma_\pi = 0.0025$$

$$\lambda = 1/16$$

- For the old regime, we calibrate the policy rule to match estimates for US data 1965-79

$$\bar{\pi} = 0.046,\ \psi_\pi = 0.043,\ \psi_y = 0.12,\ \sigma_i = 0.052.$$

S_0 is calibrated to the steady state associated with this policy.

5 The Central Bank's Decision Problem

The new governor appears at date 0.

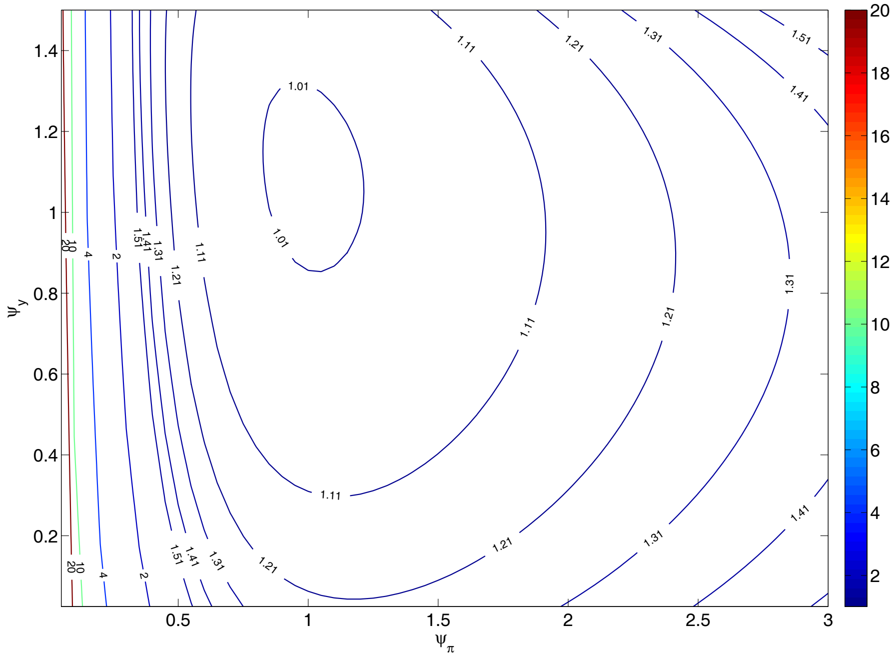
S/he arbitrarily sets $\bar{\pi} = 0.005$ and $\sigma_i = 0.0025$ (quarterly rates)

S/he chooses ψ_y, ψ_π to minimize expected loss, announcing the result along with $\bar{\pi}, \sigma_i$.

Then the public forms its prior, putting some weight (possibly zero) on the announcement and some on its experience in the old regime.

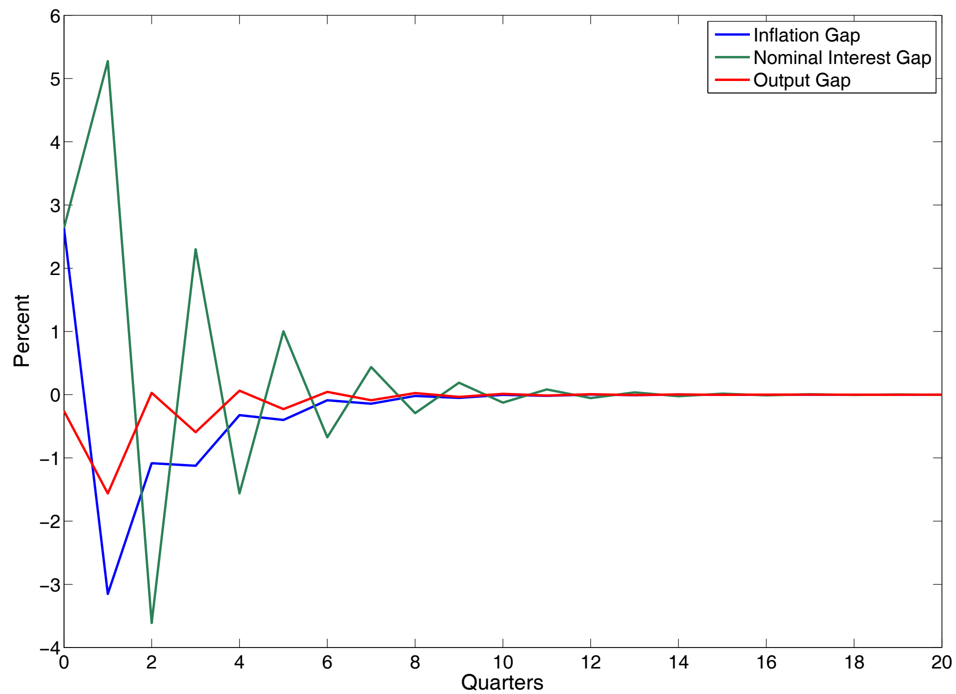
The disinflation commences at date 1.

6 Optimal Policy Under Rational Expectations



Relative loss under rational expectations for $\bar{\pi} = 2$ percent per annum

The optimal simple rule sets $\psi_\pi = 1$ and $\psi_y = 1.1$.



Transition under the RE-optimal policy

- After 8 periods, inflation is close to its new target, which is roughly 2.6 percentage points below the old target.

- The cumulative loss in output is also around 2.6 percent.

- We define the sacrifice ratio as the cumulative loss in output over 8 quarters divided by change in $\bar{\pi}$.

- Accordingly, the sacrifice ratio is approximately 1 percent of lost output per percentage point of inflation.

- The reason why the sacrifice ratio is small under RE is that the model has no indexation.

7 A Mixture Prior

Private agents form a prior by mixing two sources of information,

$$p_m(\psi) = w p_{old}(\psi) + (1 - w) p_{new}(\psi). \tag{14}$$

$p_{old}(\psi)$ encodes beliefs based on experience in the old regime

$p_{new}(\psi)$ encodes beliefs gleaned from the central bank's announcement

w is a weight assigned to the old regime

$p_{old}(\psi)$ is calibrated to match estimates of the policy rule for the period 1965-1979.

The parameters are independent a priori,

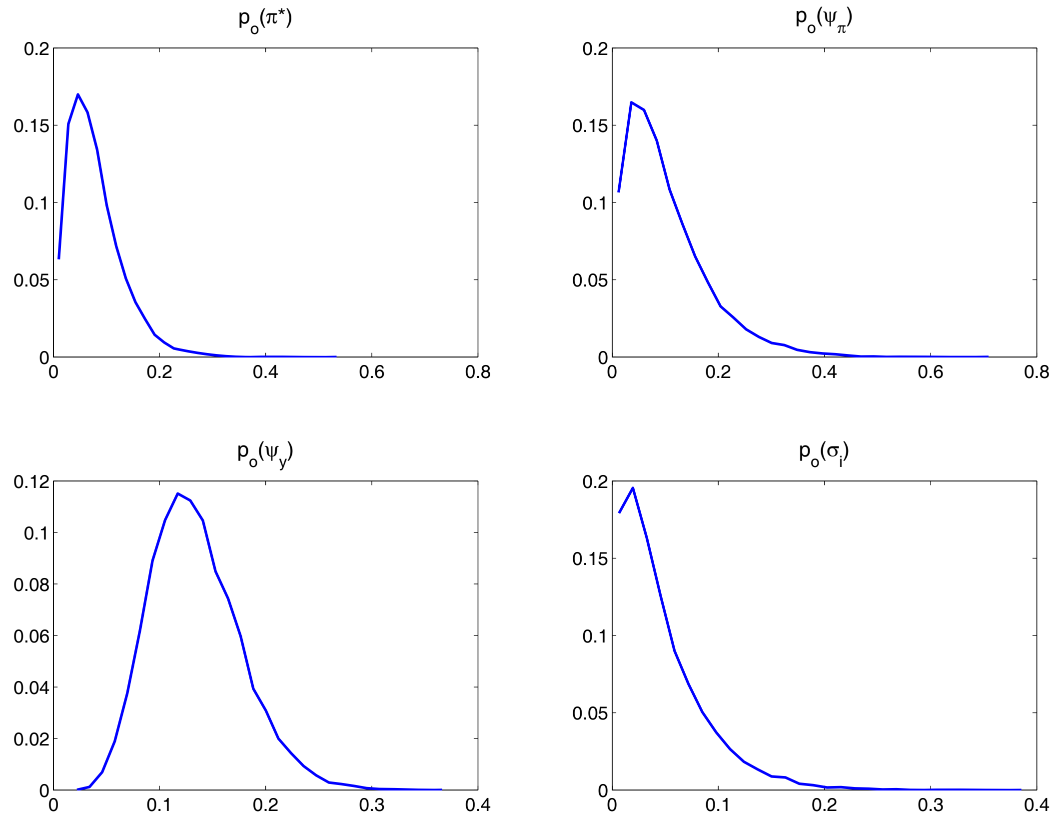
$$p_{old}(\psi) = p_{old}(\bar{\pi}) p_{old}(\psi_\pi) p_{old}(\psi_y) p_{old}(\sigma_i^2).$$

Each component is a gamma density (to enforce non-negativity).

The prior is calibrated so that its mode and variance match the OLS point estimate and its variance.

$p_{old}(\psi)$

	$\bar{\pi}$	ψ_π	ψ_y	σ_i
Mode	0.046	0.043	0.12	0.013
Standard Deviation	0.051	0.08	0.04	0.04



Experience-Based Prior

The dispersion is salient, for it signifies that the old regime lacked transparency.

$p_{new}(\psi)$ has the same functional form.

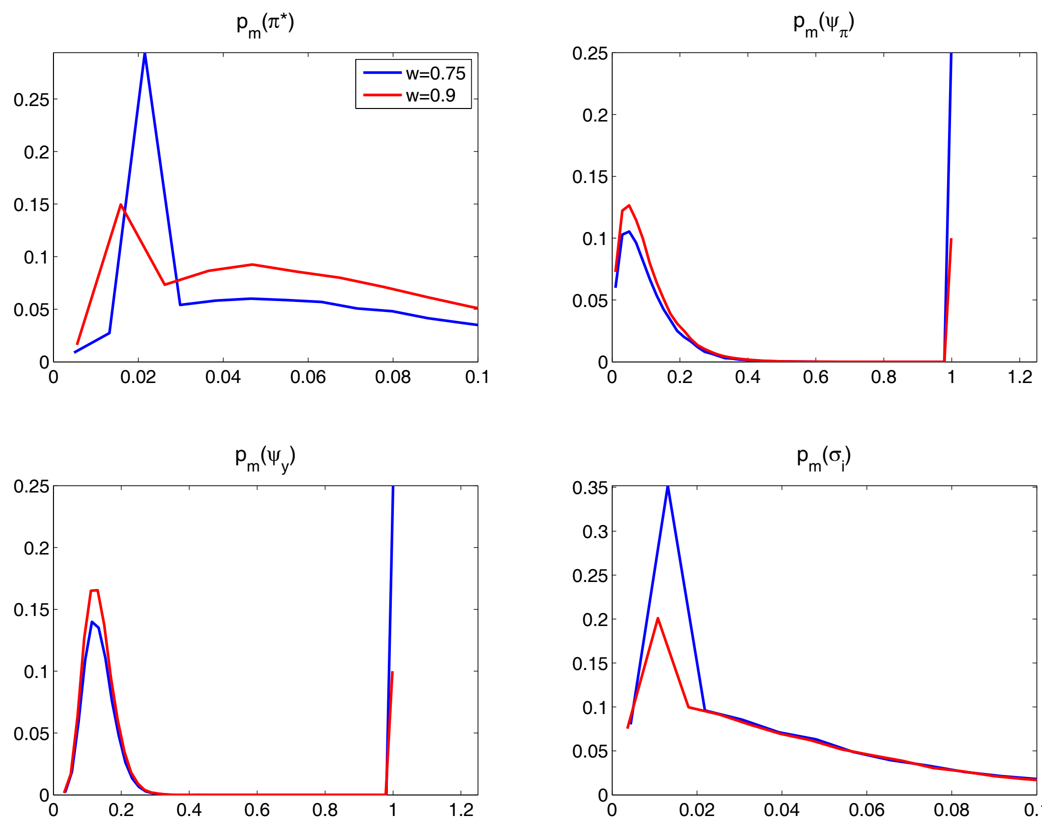
Its parameters are calibrated so that the prior is tightly centered around the bank's announcement.

I.e., although the public is skeptical that the bank will change policy (w), we assume that the public is confident that the bank will do what it says if it does switch.

We interpret the tightness of $p_{new}(\psi)$ as a measure of transparency.

An example: $\bar{\pi} = 0.02$, $\psi_\pi = 1$, $\psi_y = 1, \sigma_i = 0.01$

$w = 0.75$ and 0.9.



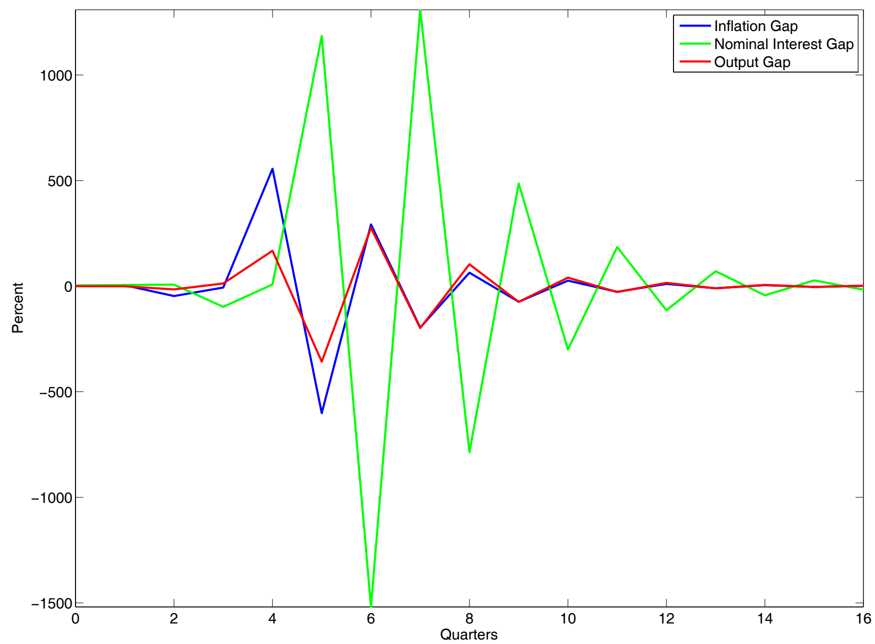
Mixture prior for $\bar{\pi} = 0.02$, $\psi_\pi = 1$, $\psi_y = 1$, and $\sigma_i = 0.01$.

- The prior for $\bar{\pi}$ has two modes, one near the old target ($\bar{\pi} = 0.046$) and another near the new ($\bar{\pi} = 0.02$).

- The announcement can be influential even when experience is weighed more heavily.

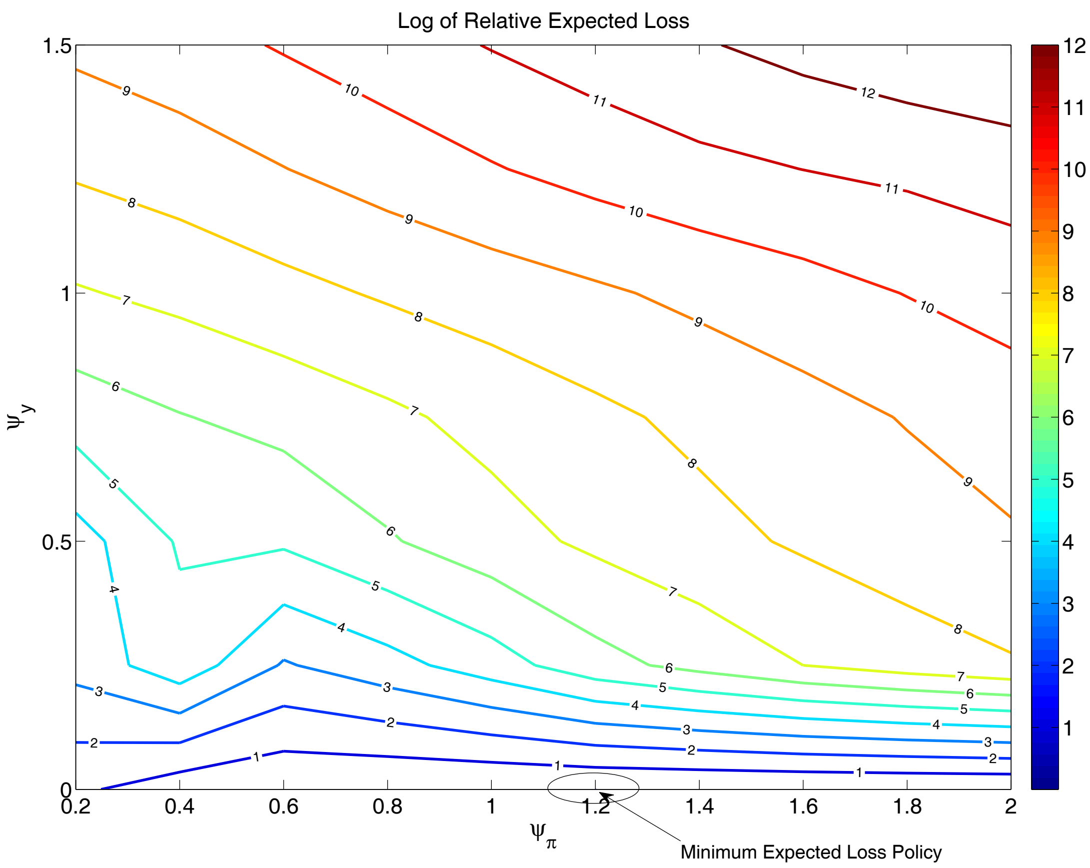
- This happens because $p_{new}(\psi)$ is more tightly concentrated than $p_{old}(\psi)$.

8 Outcomes under learning when $w = 1$



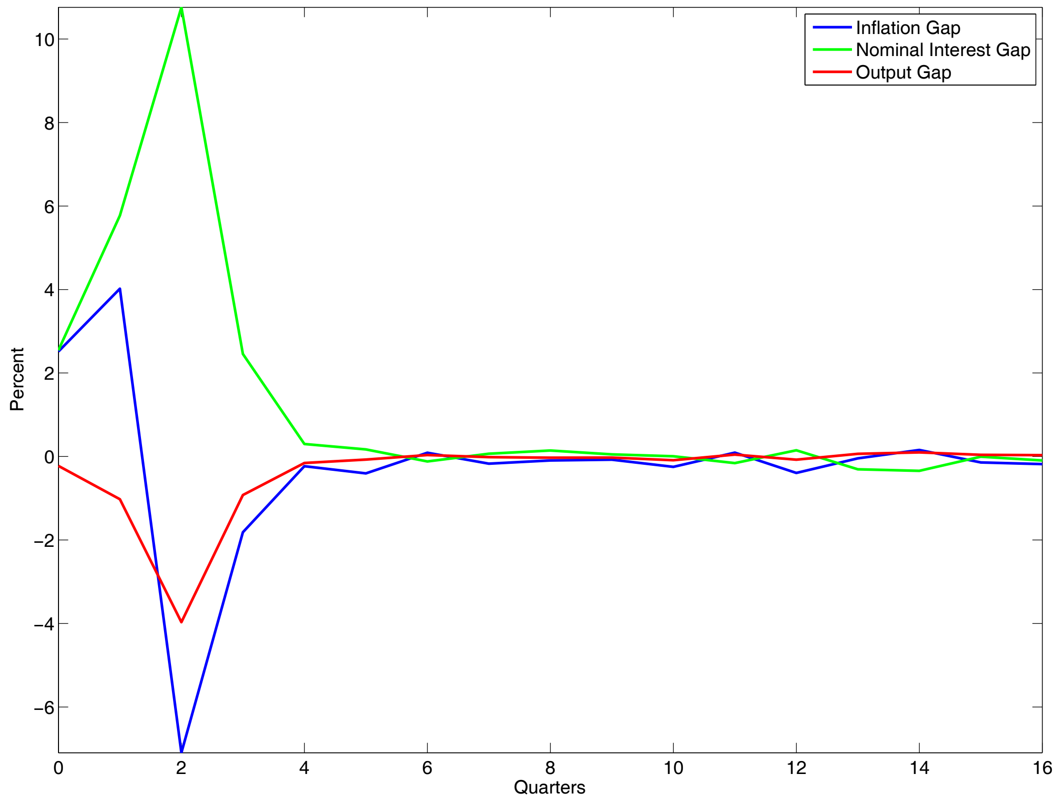
Average responses under the RE-optimal policy.

The RE-optimal rule generates catastrophic losses.



Isoclines for log relative expected loss.

Expected loss is minimized by setting $\psi_\pi = 1.2$ and $\psi_y = 0$.

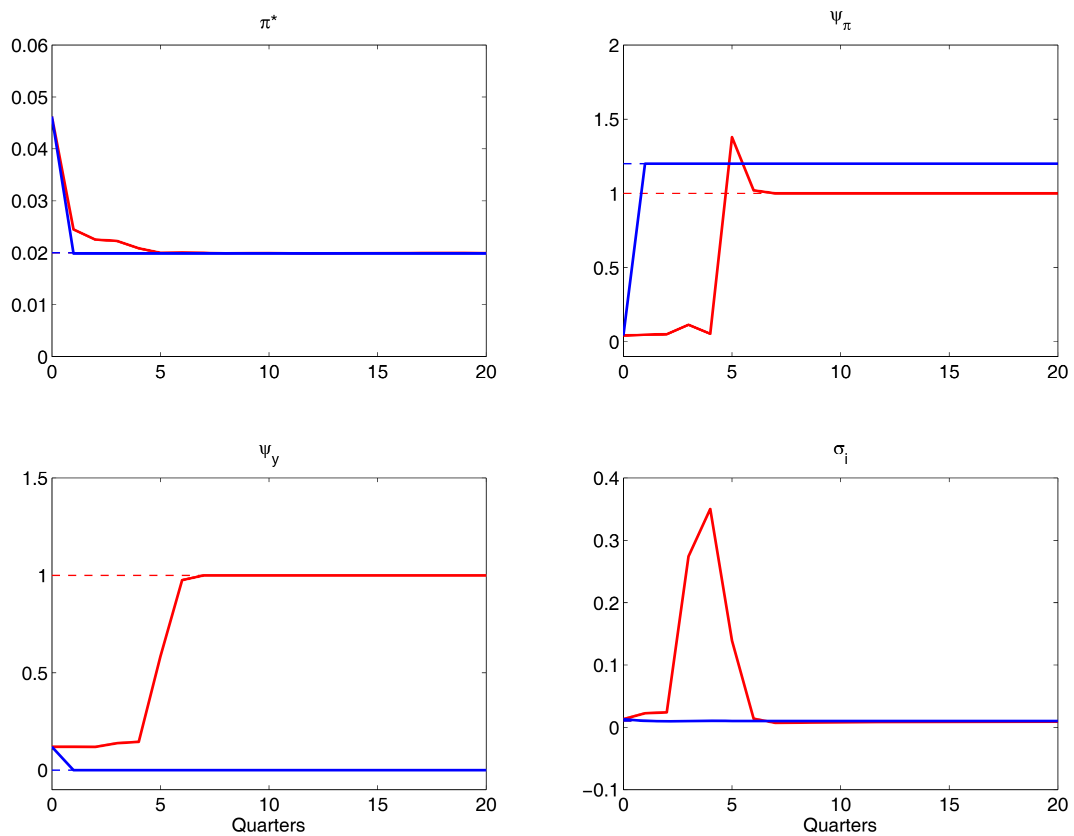


The transition under the minimum-expected-loss policy

After 8 quarters, inflation has declined permanently by 2.6 percentage points at the cost of a cumulative loss in output of 6.4 percent of a year's GDP.

This implies a sacrifice ratio of 2.5 percentage points of lost output per point of inflation.

Higher than under RE, but small relative to the predictions of Keynesian macro-econometric models of the 1970s.



Average of posterior mode estimates

Learning is slower under the RE-optimal policy.

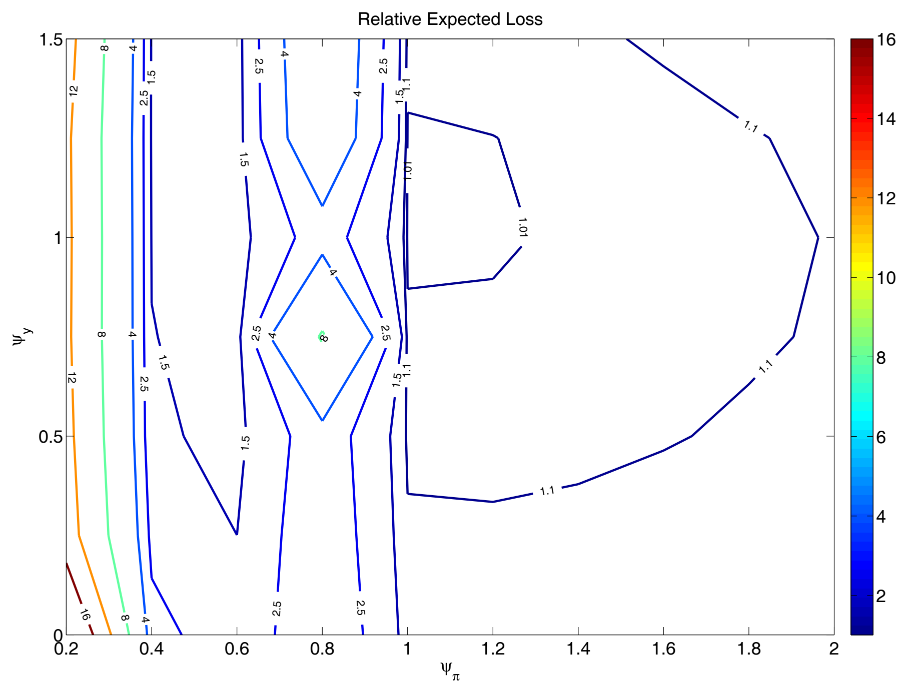
- Estimates of ψ_π and ψ_y change little in the first year.
- Because beliefs about ψ_π and ψ_y are essentially unchanged, movements in i_t are mostly unexpected and perceived as shocks.
- Estimates of σ_i therefore increase dramatically.
- This makes inflation and output gaps highly volatile.
- That variation identifies ψ_π and ψ_y by the middle of year 2/
- By then the economy is oscillating wildly, and the central bank is playing catch up.

Learning is faster under the minimum expected loss policy

- Estimates converge to the true policy coefficients at the end of the first period.

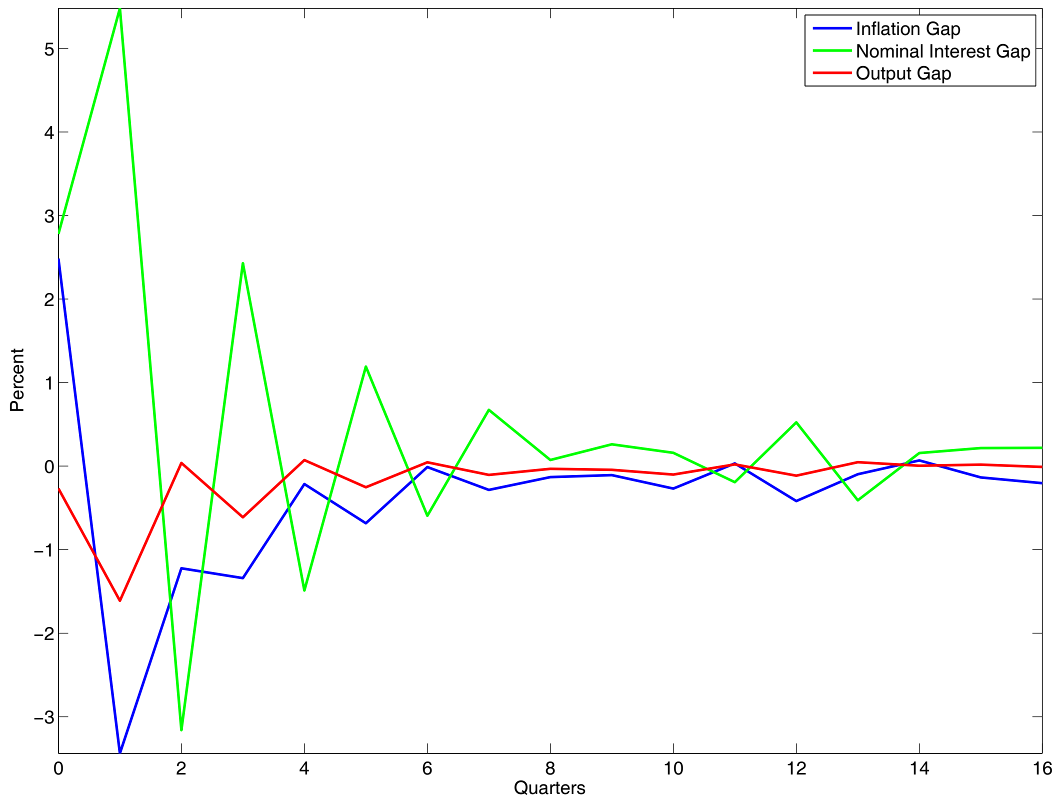
- From that time forward, the economy behaves essentially as a rational-expectations economy.

9 Outcomes when $w = 0.75$

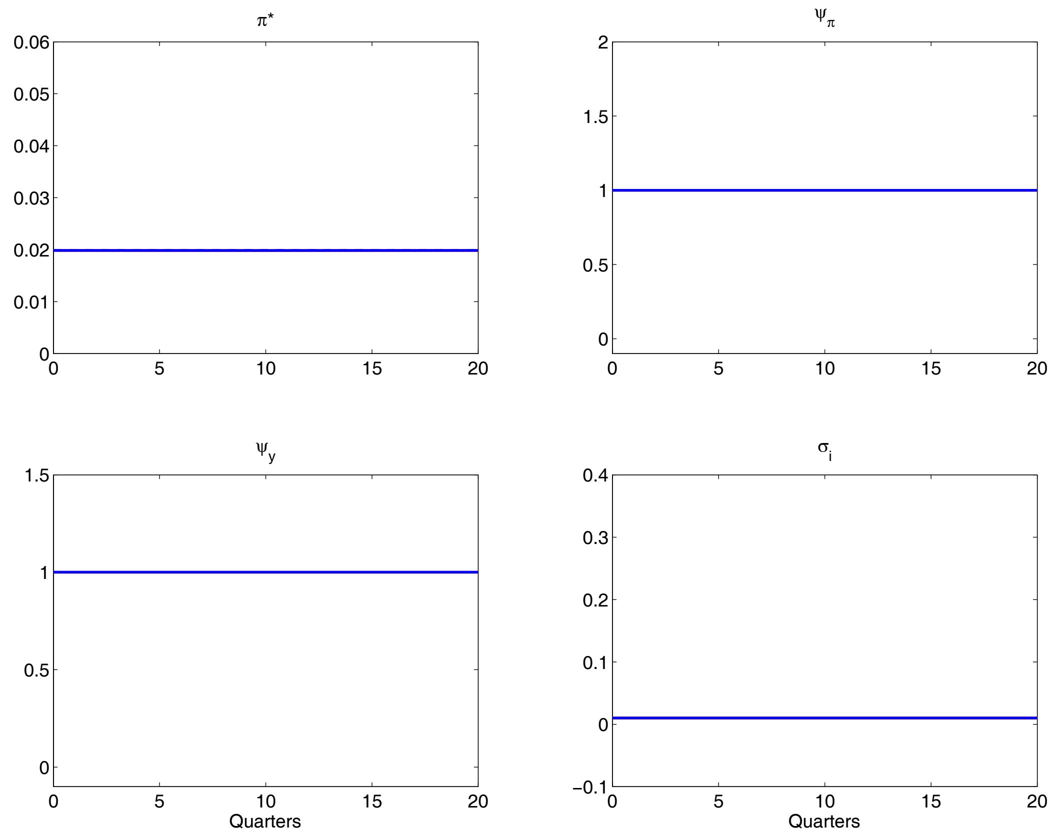


Isoclines for relative expected loss when $w = 0.75$.

Expected loss is minimized by setting $\psi_\pi = \psi_y = 1$, which approximates the RE-optimal rule.



The transition under the minimum-expected loss policy

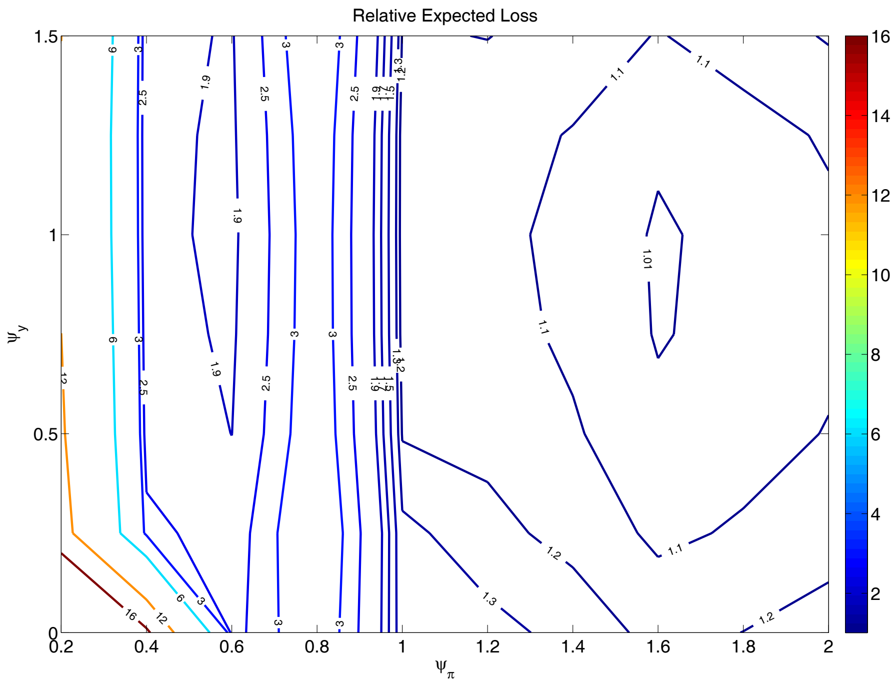


Average of posterior mode estimates

Enough weight is placed on the announcement to move the posterior mode to the true coefficients at date 0.

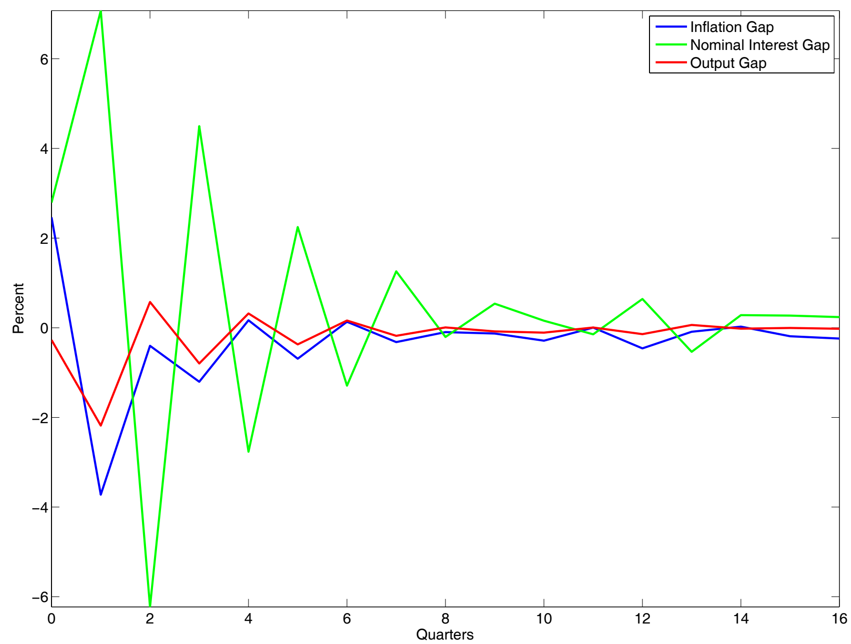
Hence the optimal policy and transition resemble that under RE.

10 Outcomes when $w = 0.9$

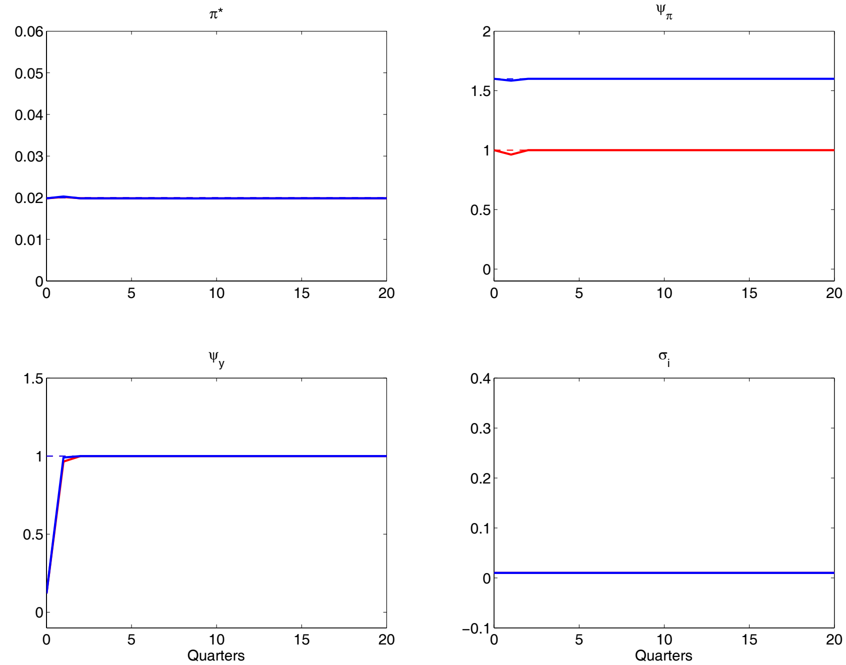


Isoclines for relative expected loss when $w = 0.9$.

Expected loss is minimized by setting $\psi_\pi = 1.6$, $\psi_y = 1$.



Transition under the minimum-expected-loss policy.



Average of posterior mode estimates

11 Conclusion

Some policies that work well under rational expectations generate catastrophic losses under learning.

But some good policies exist that generate rapid transitions with little loss.

The central bank can obtain results close to those of Sargent even when credibility is far from perfect.

Transparency of the new regime is critical for getting there, for that accelerates learning.

12 Additional Slides

$$A_t = \begin{bmatrix} 1 & 0 & -\varsigma_t & -1 & -\kappa_t & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\lambda_{1t} & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$B_t = \begin{bmatrix} \beta_t + \gamma_{1t}(\theta - 1) & \gamma_{1t} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \gamma_{2t}(\theta - 1) & \gamma_{2t} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$C_t = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu_{\pi t} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu_{\phi t} \\ 0 & 0 & \lambda_{2t} & 0 & 0 & 0 & 0 & 0 & \mu_{\delta t} \\ 0 & 0 & 0 & \rho_u & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu_y \\ 0 & 0 & 0 & 0 & 0 & \rho_g & 0 & 0 & \mu_g \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ \psi_{\pi t} & 0 & 0 & 0 & \psi_{yt} & 0 & -\psi_{yt} & 1 & -\psi_{\pi t}\bar{\pi}_t \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\begin{aligned} \mu_{\pi t} &= [1 - \beta_t - \gamma_{1t}(\theta - 1)]\bar{\pi}_t - \kappa_t \overline{y}_t - \varsigma_t \overline{\delta}_t, \\ \mu_{\phi_t} &= -\gamma_{2t}(\theta - 1)\bar{\pi}_t \\ \mu_{\delta t} &= (1 - \lambda_{2t})\,\overline{\delta}_{t-1} - \lambda_{1t}\bar{\pi}_t \\ \mu_y &= r - g \\ \mu_g &= \left(1 - \rho_g\right) g. \end{aligned} \tag{15}$$

$$D_t = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

$$C_{at} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu_\pi \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu_\phi \\ 0 & 0 & \lambda_2 & 0 & 0 & 0 & 0 & 0 & \mu_\delta \\ 0 & 0 & 0 & \rho_u & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu_y \\ 0 & 0 & 0 & 0 & 0 & \rho_g & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ \psi_\pi & 0 & 0 & 0 & \psi_y & 0 & -\psi_y & 1 & -\psi_\pi \bar{\pi} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$