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Fiscal Shocks and Fiscal Risk Management *

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Résumé:
Nous utilisons les rendements de plusieurs actifs financiers internationaux pour identifier les chocs exogènes au surplus fédéral canadien. Nous trouvons qu'une grande proportion de la variation du surplus peut être répliquée par une combinaison linéaire de ces rendements et que la dette croissante observée durant les années 1980 et 1990 était le résultat de chocs exogènes négatifs et d'une réponse retardée du gouvernement face à ces chocs. Nous développons un cadre formel permettant d'évaluer les gains potentiels provenant d'une stratégie de gestion du risque fiscal utilisant ces actifs pour se couvrir contre des chocs exogènes. Nous montrons que la gestion du risque fiscal peut générer des gains en bien-être significatifs en améliorant la soutenabilité de la politique fiscale et ainsi en réduisant les taux d'imposition moyens.

Abstract:
We use the returns on a set of international financial securities to identify exogenous shocks to the Canadian federal surplus. We find that a large portion of the variation in the surplus can be replicated by a linear combination of these returns and that the rising debt observed in the 1980s and 1990s was a result of adverse exogenous shocks and a delayed response by the government to these shocks. We develop a formal framework to evaluate the potential gains from a fiscal risk management strategy, using these securities to hedge against exogenous shocks. We show that fiscal risk management can generate significant welfare gains by enhancing the sustainability of fiscal policy and thereby lowering average tax rates.

Keywords:
Fiscal policy, sustainability, asset pricing, risk management

JEL classification: E6, F3, H6

1. Introduction

The large public debts accumulated by many OECD economies during the last two decades have created serious concern about the sustainability of fiscal policies and have become a major topic of policy debate. Much of the economic literature on this subject rationalizes these large public debts as the consequence of public sector bias towards deficits.[1] However, the analysis fails to explain why the public debt problem emerged in the mid–seventies and not before (see Alesina and Perotti, 1995). To address this question, some authors have started to consider the role of exogenous "fiscal shocks" — unexpected changes in government spending and revenues — as a source of the rising public debt. However, their attention has largely been focused on how political and fiscal institutions affect the government's response to fiscal shocks,[2] rather than on the shocks themselves. Few studies have tried to investigate the actual contribution of fiscal shocks to the rising public debt. The question of what governments can and should do to mitigate these shocks is also left unanswered.

The focus of this paper is on fiscal shocks and it contributes to the literature in three key ways. First, we estimate the contribution of exogenous factors to variations in the primary surplus and measure their empirical importance to the rising public debt. Second, we characterize the response of fiscal policy to these shocks. Finally, we show how a policy of fiscal risk management can mitigate the adverse consequences of these external influences, and estimate the potential gains from fiscal risk management in terms of improved sustainability, lower average taxes and increased welfare. As an empirical example we focus on the primary surplus and debt levels of the Canadian federal government over the last forty years. Canada provides a good example for our analysis because, during this period, the behavior of the public debt in Canada was very similar to that of the OECD average. Moreover, since Canada is a small open economy, it is much easier to identify fiscal shocks that are clearly exogenous to the Canadian government.

A key novelty of our analysis is that we identify exogenous fiscal shocks using the returns on several international financial securities. The idea behind this approach is that if the financial market is relatively complete, then the relevant risk can be represented by some combination of these market returns.[3] In addition to being exogenous with respect to Canada's fiscal policy, we find that these variables capture a larger portion of the variation in the primary surplus than the growth rate of

[1] See Persson and Svensson (1989), Roubini and Sachs (1989), Tabellini and Alesina (1990), Grilli, Masciandaro and Tabellini (1991), Tabellini (1991), Von Hagen (1992) and Alesina and Perotti (1996). Alesina and Perotti (1995) provides an excellent survey of this literature.

[2] See Von Hagen (1991), Alt and Lowry (1994), Poterba (1994), Bayoumi and Eichengreen (1995), Bohn and Inman (1995) and Alesina and Perroti (1996).

[3] This is referred to in the finance literature as the "spanning property".

GDP or the unemployment rate — variables that have been used in previous studies (e.g. Roubini and Sachs, 1989). Indeed, our empirical analysis reveals that the majority of the variation in the primary surplus over this period can be replicated by a linear combination of returns on international financial securities. In particular, the large deficits that were experienced in the seventies and early eighties can largely be attributed to these exogenous fiscal shocks. We also find that the surplus process is best characterized by a time–invariant function of current and past shocks with an abrupt policy regime shift towards higher primary surplus in the mid–1980s.

Our empirical results suggest that the problem of rising public debt in Canada was caused by a series of adverse exogenous shocks that occurred in the late seventies and early eighties, and that the problem was aggravated by the delay in the government's response to the rising debt. Rather than adjusting the primary surplus continually in response to the rising debt level, the Canadian fiscal authorities maintained the original fiscal policy rule long after the adverse shocks occurred. A significant shift in the stance of fiscal policy took place only when the net debt reached an alarmingly high level and a new government came into power.[4] Our evidence appears to be more consistent with the predictions of a political economy model (e.g. Alesina and Drazen, 1991) that emphasizes the role of adjustment costs in causing delays, rather than with the basic tax–smoothing model.

The use of financial market returns to represent exogenous shocks to the primary surplus allows us to address two key questions: (1) To what extent can exogenous shocks to the primary surplus be diversified in the international financial market, and (2) What are the potential gains of adopting such a fiscal risk management strategy? According to the optimal dynamic taxation theories of Barro (1979) and Lucas and Stokey (1983), tax rates should be maintained at relatively constant levels and should not be used to offset all of the exogenous shocks to the primary surplus. For political and institutional reasons, fiscal authorities may not be able to adjust fiscal policy instantaneously. In the absence of state contingent borrowing and lending, however, a stable fiscal policy may become unsustainable as the effects of the exogenous shocks accumulate and result in a rising debt,[5] which could force the government to drastically raise taxes and cut spending in order to reduce the debt. The more volatile is the primary surplus, the more likely it is that the tax rate will have to be increased in the future. By hedging away the volatile component of the primary surplus that is associated with the exogenous shocks, fiscal risk management might help to reduce the probability of an excessively large and rising public debt.

[4] The year in which the fiscal policy change took place is also the year after the Conservative Party became the majority party in the Canadian parliament.

[5] Bohn (1991) provides several theoretical examples that illustrate the need for the government to issue state contingent bonds in stochastic economies.

To address these issues, we develop a conceptual framework that is consistent with our empirical observations and consider a simple hedging strategy that effectively replaces the diversifiable component of the primary surplus with a constant cash–flow that has the same present value. We estimate the gains from fiscal risk management in terms of the sustainability of fiscal policy, average tax rates and welfare. We find that by increasing the sustainability of fiscal policy, fiscal risk management results in sizable welfare gains by lowering expected tax rates in the short and long run.

We are not the first to emphasize the importance and potential benefits of using financial market instruments in government finance. In their seminal work, Lucas and Stokey (1983) show that state contingent bonds are crucial in implementing the optimal fiscal policy and maintaining the time–consistency of the policy in the face of stochastic shocks.[6] Bohn (1990) provides empirical evidence that financial market instruments may help the US government to further smooth its tax rates and, therefore, improve welfare. In contrast, we emphasize the role of fiscal shocks in affecting the variation in budget surplus rather than in the tax rates. Our analysis shows that the impacts of these shocks have largely been absorbed by the government through risk–free borrowing and lending and have, therefore, resulted in rising debt. While Bohn argues that risk management may help the government to reduce the short–run variations in the tax rates, we show that fiscal risk management can also help to reduce the probability of having a high debt, enhancing the long–run stability of fiscal policy and thereby raise welfare.

The analysis contained in the rest of the paper is divided into two sections. In Section 2 we estimate the impact of exogenous factors on the surplus process and determine the response of the fiscal policy stance of the government to those factors. In Section 3 we develop a method for evaluating the potential gains of fiscal risk management, and apply it to the Canadian example using the empirical specification developed in Section 2. Section 4 offers some concluding remarks. Details regarding data and our calibration methodology are given in Appendix A. Other details regarding the robustness of our empirical results and technical details of our simulation algorithm are provided in Appendix B.[7]

2. Fiscal Shocks and Policy Response

After being consistently in surplus until the early 1970s, the Canadian budget balance exhibited

[6] Zhu (1995) shows that, in an economy with capital, government bonds that are linked to capital returns can be used to ensure the time–consistency of optimal fiscal policy even if there are no stochastic shocks.

[7] Appendix B is available at http://www.chass.utoronto.ca/~lloydell/papers/fiscrisk.html.

almost continuous deficits during the seventies and early eighties, followed by persistent surpluses from the mid–1980s. The purpose of this section is to determine how much of the variation in the surplus can be attributed to external influences and what parts to significant shifts in "fiscal stance".

2.1 The Exogenous Fiscal Shocks

We use the market returns on a set of international financial assets to measure fiscal shocks. These market returns have been used extensively in the finance literature to represent underlying factors in stock market returns and to capture cyclical activity in the US (and, hence, the Canadian) economy. Since Canada is a small open economy, it is safe to assume that these international variables are not influenced by the government's fiscal policy. Moreover, if the financial markets are relatively complete, then it should be possible to replicate a large portion of the fiscal risk using some combination of these returns.

The asset return variables are the value weighted return on the New York Stock Exchanges VWR (from the CRSP tape), the dividend yield DIV on the CRSP value–weighted index (measured as a 1–year backward moving average of dividends divided by the most recent stock price), the 3–month Treasury bill rate TBILL, the 1 year moving average of the 3–month Treasury bill rate TBMA; and the yield on 10 year government bonds, LONGR. These variables, or linear combinations of them, have been found to forecast asset returns and are discussed in more detail in Campbell (1996). We multiply each of these by the nominal exchange rate to obtain the Canadian dollar value of the return on each US dollar invested. This ensures that the US dollar values of the returns are independent of the exchange rate and, therefore, Canadian government's policy. Let X_t denote the vector that contains these return variables.

2.2 The Fiscal Policy Rule

We adopt the following specification of the primary surplus process:

$$s_t = \Gamma_t + \mathbf{a}' X_t + \rho s_{t-1} + \varepsilon_t. \tag{1}$$

Here s_t is the ratio of the nominal primary surplus to nominal GDP. The vector $\mathbf{a}$ measures the marginal impact of the exogenous shocks to the primary surplus. It represents both the effects of the shocks under a given policy and the effects through policy response to shocks.[8] The term Γ_t summarizes the permanent components of the government's policy variables. It could be time–

[8] For example, government expenditures on existing programs may be a function of the shocks, $\mathbf{a_0}' X_t$, and the government responses to the shocks by setting the taxes equal to $\mathbf{a_1}' X_t$. Then, $\mathbf{a} = \mathbf{a_0} + \mathbf{a_1}$. In principle, these parameters may not be constant over time.

varying and may change in response to the debt level and political events. The lagged surplus term is intended to capture the persistency of the surplus process.[9] Finally, the error term ε_t is assumed to be a normal random variable that is uncorrelated with X_t, Γ_t and s_{t-1}. In section 3, we present a small open economy model in which the equilibrium primary surplus process takes exactly the form that is specified in (1) with $\rho = 0$, and Γ_t is a linear function of the effective tax rate on output.

In order to estimate (1), we need to specify how the policy variable Γ_t evolves over time. Two alternative specifications are considered in our empirical analysis: 1) Γ_t is a linear function of debt–GDP ratio, and 2) Γ_t is a step function of time. The first case corresponds to Barro's tax smoothing policy according to which the permanent components of the tax policy should be adjusted continually in response to the debt level. The policy rule implied by the second case is consistent with the recent political economy literature that emphasizes delayed fiscal adjustments due to political or institutional constraints.

2.3 The Contribution of Fiscal Shocks to the Primary Surplus

To determine the quantitative importance of exogenous shocks to the variation of the primary surplus, we first run a simple linear regression of the surplus–GDP ratio, s_t on the shock variables X_t over the period 1958:1 1994:4. The result is reported in the first column of Table 1. Although the Durbin–Watson statistic suggests that there is serial correlation in the residuals, this regression illustrates the striking fact that almost 70% of the variation in the surplus can be replicated by a simple linear combination of the asset returns. When we include a lagged dependent variable, as in the second column, the specification of the model improves, but it does not add much in terms of its explanatory power. Moreover, the parameters of the model are quite robust to its inclusion.[10]

Figure 1 shows that there appears to have been a shift in the mean of the residuals during the mid–1980s. After this point, although the model continues to replicate the direction of movements in the surplus, it understates its true level. This is consistent with a level shift in the permanent components of the government's fiscal policy. The CUSUM test (which is a t–statistic testing for structural stability) reported in the table shows that the null of no structural change is rejected at the 5% level.

— FIGURE 1 —

[9] This could be due to the costs associated with adjusting the surplus to the government's long–run target levels.

[10] As a comparison, we also ran the same regressions but including the change in real GDP growth and the unemployment rate, instead of X_t, as shock variables. The results show that a much smaller portion of the variation in the surplus can be explained by these traditional shock variables (see Appendix B).

2.4 The Shift in Fiscal Stance

There are several reasons to suspect that there may have been a significant change in the fiscal stance of the Canadian government during the 1980s. These include the rapidly rising debt, the associated pressure from financial markets and a shift to a more conservative role for government. In the third column of Table 1, we include the debt–GDP ratio (DEBT) as an additional regressor to see if the structural change(s) can be explained by a continuous response of the fiscal policy to the rising debt, as suggested by the Barro's tax smoothing theory. Although DEBT is indeed a statistically significant regressor, there is still evidence of structural change as indicated by the CUSUM test.

An alternative to Barro's tax smoothing theory is the war of attrition model of Alesina and Drazen (1991), which emphasizes the political costs of adjusting fiscal policy. According to this theory, fiscal response to the rising debt may be delayed due to conflicts among different groups about how the burden of the required policy change is to be shared. Because of the delay, the change in the government's fiscal policy is better described by a discrete regime change. Here we identify the potential regime change by estimating the following switching regression model:

$$s_t = \begin{cases} c_1 + \mathbf{a}_1' X_t + \rho_1 s_{t-1} + \varepsilon_{1t}, & \text{if } t \leq t^*; \\ c_2 + \mathbf{a}_2' X_t + \rho_2 s_{t-1} + \varepsilon_{2t}, & \text{if } t > t^*. \end{cases} \quad (2)$$

where $\varepsilon_{1t} \sim N(0, \sigma_1^2)$, $\varepsilon_{2t} \sim N(0, \sigma_2^2)$ and t^* is unknown. We use the maximum likelihood method to estimate both the parameters and the break point t^*. The estimated t^* is the second quarter of 1986. Figure 2 illustrates the maximized log–likelihood function (conditional on t^*) for different switching dates. This figure illustrates quite clearly why maximum likelihood estimation pinpoints the structural break as having occurred between the first and second quarter of 1986. We test the significance of a structural break at this date with Chow tests for each of the regressions models discussed above. As can be seen this hypothesis cannot be rejected at the 5% level.

— FIGURE 2 —

To examine further the nature of this regime switch we re–run the regression reported in column 3 of Table 1 by introducing a dummy variable which takes on the value one after 1986:1 and zero otherwise. The fourth column of Table 1 documents this regression. When we account for the structural break in this way, the debt–GDP ratio is no longer a significant explanatory variable. This suggests that its significance in the previous regression was not the result of a stable relationship over shorter sub–periods. Indeed, when we estimated the regression within each regime, the debt–GDP ratio was no longer significant.[11] Our preferred model is therefore represented by the regression model documented in the last column of Table 1. It corresponds to a special case of (1) when f_t is a step function of time that has an upward step at the second quarter of 1986.

To test the stability of the parameters on the shocks and the lagged surplus across the two regimes, we also regressed the residuals from our preferred regression on the explanatory variables within each regime. We could not reject the joint hypothesis that these parameters are constant across regimes. In other words, the change in policy stance is largely consistent with an increase in the permanent components of the surplus after the first quarter of 1986 rather than a change in the marginal responsiveness of the surplus to the exogenous shocks.[12]

— TABLE 1 —

Based on these empirical results, we interpret the Canadian government's surplus process as follows: Under the fiscal policy rule that was in place in the 1960s and 1970s, the exogenous fiscal shocks accounted for about 70% of the variation in the primary surplus. Until the mid–1970s, the combination of the policy rule and the shocks had resulted in positive surpluses on average. Beginning in the mid–1970s, however, the exogenous shocks caused a sustained period of deficits and resulted in the rising debt under the original policy rule. Instead of adjusting its fiscal policy immediately in response to the deficits and rising debt, the government continued the original policy until 1985 when it adjusted the surplus level upward permanently. This adjustment, along with more favorable exogenous shocks, resulted in a return to positive primary surpluses in the late 1980s and early 1990s. The behavior of the Canadian primary surplus therefore appears to be more consistent with the political economy model of Alesina and Drazen (1991) than with the tax smoothing model of Barro (1979).

3. Fiscal Risk Management

The analysis of the previous section raises some intriguing questions: Given that much of the variation in the surplus can be replicated by the return on a portfolio of international securities, could the government mitigate the impact of these fiscal shocks by hedging the risk? Moreover, under what conditions would such a policy be desirable? In this section, we investigate the potential role

[11] The reason for the significance we found in the previous regression seems to stem from the fact that both the surplus and the debt has moved upward during 1980s.

[12] As we show below, a forecast of the surplus in the latter half of the sample using data from the first half of the sample tracks the actual surplus quite well (see Figure 3).

for systematic fiscal risk management as part of the government's overall debt policy. To do so, we develop a framework for analyzing the impact of the government's fiscal policy on welfare that is consistent with the international asset pricing model that we use for determining the cost of hedging.

3.1 A Small Open Economy

Consider a small open economy populated with a large number of identical, infinitely–lived households. There is only one tradable physical good that is consumed by both domestic and foreign households. Thus, the purchasing power parity holds in this model economy. Time is discrete. Let z_t be the state variable that summarizes the exogenous shocks in period t, and $z^t = (z_0, z_1, ...z_t)$ be the history of shocks up to period t. We assume that there is a complete world financial market in which all contingent claims with payoffs that are a measurable function of $z^{(t)}$ can be traded. Under this assumption and the assumption of no–arbitrage, there exists a unique sequence of stochastic discount factors, $\{M_t\}_{t\succeq 0}$, such that the time t price of a contingent claim that pays $f(z^{(t+j)})$ units of the consumption good in period $t+j$ is[13]

$$\Pi^f(t,j) = E_t\left[\frac{M_{t+j}}{M_t} f(z^{(t+j)})\right] \tag{3}$$

The small open economy assumption implies that the stochastic discount factors are exogenous with respect to domestic agents' actions. In particular, changes in the domestic government's fiscal policy has no effect on them.

Preference and Technology

The domestic households' preference over the consumption plan $\underline{c} = \{c_t\}_{t\succeq 0}$ is represented by a utility function $U(\underline{c})$. Here, we do not impose any restrictions on $U(.)$ except that it is concave and that, for any $\underline{c}$ and $\delta > 0$, $U(\underline{c}+\delta) > U(\underline{c})$, where $\underline{c}+\delta$ is defined as $\{c_t+\delta\}_{t\succeq 0}$. In particular, $U(.)$ can be the same or different from those of the foreign investors and it can represent non–expected utility, as in Epstein and Zin (1989), or time non–separable utility, as in Abel (1990). Our welfare measure defined below is independent of these considerations.

The representative household in the domestic economy can produce the consumption good with the following concave production technology:

$$y_t = \psi_t k_t^\alpha, \quad 0 < \alpha < 1, \tag{4}$$

where y_t and k_t are the period t output and capital input, respectively, and ψ_t is domestic productivity,

[13] See, for example, Harrison and Kreps (1979).

which evolves stochastically. The output can be either consumed or converted one to one into capital. The capital investment decision is made one period ahead and capital depreciates at the rate δ. The government taxes output minus depreciation at the rate τ_t. In each period, the household allocates her after–tax output between consumption, capital investment and investment in financial contingent claims. Let c_t and $b_{t+1}(z^{(t)}, z_{t+1})$ denote the household's consumption and contingent claim holding in period t, respectively. Then, the household's budget constraint in period t is

$$c_t + E_t\left[\frac{M_{t+1}}{M_t} b_{t+1}(z^{(t)}, z_{t+1})\right] + k_{t+1} \leq (1-\tau_t)(\psi_t k_t^{\alpha} - \delta k_t) + k_t + b_t. \tag{5}$$

Optimal Investment and the Welfare Function

Under the complete market assumption, Fisher separation applies with respect to the household's consumption and production decisions. Let

$$W_0 = \frac{1}{M_0}\sum_{t=0}^{\infty} E_0 M_t \left[(1-\tau_t)\left(\psi_t k_t^{\alpha} - \delta k_t\right) + k_t - k_{t+1}\right]. \tag{6}$$

denote the household's wealth at time zero. Then, for any given level of W_0, the household's welfare is

$$V_0(W_0) = \max_{\underline{c}}\{U(\underline{c}),\ \text{s.t. } \frac{1}{M_0}\sum_{t=0}^{\infty} E_0 M_t c_t \leq W_0\}. \tag{7}$$

In particular, the government's fiscal policy affects the household's welfare only through its impact on current wealth W_0. Thus, from now on, we will refer to W_0 as the household's welfare. Since this welfare measure is independent of preferences, any welfare gains from fiscal risk management must come from increasing the domestic household's wealth, *not* from smoothing the household's consumption profile.

For any given fiscal policy, the household chooses her investment decision to maximize her production wealth, W_0. It is straightforward to show that the optimal investment rule is given by

$$\hat{k}_t = \left(\frac{\alpha E_{t-1}\left[\frac{M_t}{M_{t-1}}(1-\tau_t)\psi_t\right]}{1 - E_{t-1}\left[\frac{M_t}{M_{t-1}}(1-\delta(1-\tau_t))\right]}\right)^{\frac{1}{1-\alpha}}, \tag{8}$$

so that the household's maximized wealth is

$$\hat{W}_0 = (1-\delta(1-\tau_0))k_0 + (1-\alpha)\frac{1}{M_0}\sum_{t=0}^{\infty} E_0\left[M_t(1-\tau_t)\psi_t \hat{k}_t^{\alpha}\right]. \tag{9}$$

Given a joint distribution over future fiscal policy and the stochastic discount factor, (9) can be used to compute aggregate expected wealth. With complete markets, changes in wealth provide a compensating variation measure of welfare that is independent of domestic preferences.

3.2 The Government's Fiscal Policy

The government's primary surplus in period t is

$$S_t = \tau_t \psi_t \hat{k}_t^{\alpha} - G_t(z_t). \tag{10}$$

where G_t denotes government expenditure. Let $s_t = S_t/y_t$, and $g_t = G_t/y_t$. We assume that $\{g_t\}_{t \succeq 0}$ is an exogenous stochastic process. The implied primary surplus–output ratio can then be expressed as

$$s_t = \tau_t - g_t(z_t). \tag{11}$$

The expression of the equilibrium primary surplus process in (11) is consistent with the empirical specification we used in section 2 with $\rho = 0$. In fact, if we assume that

$$g(z_t) = \overline{g} - \mathbf{a}'\mathbf{X}_t - \varepsilon_t, \tag{12}$$

where $\overline{g}$ denotes the mean government spending to output ratio, it follows that the permanent component of the government's policy variables (the first term in (1)) can be represented as

$$\Gamma_t = \tau_t - \overline{g}. \tag{13}$$

Thus, a change in Γ_t corresponds to a change in the tax rate τ_t.

If the government could issue state–contingent debt, then it is easy to show that the optimal tax policy would imply a constant tax rate across time and states. However, if the government can only issue nominal risk–free debt, a constant tax rate may result in a debt path that is unsustainable. To ensure sustainability in this case, a policy rule must be specified to determine how the tax policy will change in response to the rising debt. In the following, we consider two policy rules that are consistent with the empirical behavior of the primary surplus we reported in section 2.

Benchmark policy rule

Let $D_t(\tau)$ be the real level of government debt at the end of period t when the effective tax rate in period t is τ. Then, we have

$$D_t(\tau) = (1 + r_{t-1})D_{t-1} - [\tau - g_t(z_t)]y_t. \tag{14}$$

where r_t is the effective real interest rate on government debt. We also define

$$V_t(\tau) = \frac{1}{M_t} \sum_{j=1}^{\infty} E_t M_{t+j}[\tau - g_{t+j}(z_{t+j})] y_{t+j} \tag{15}$$

to be the present value of the government's real primary surpluses if the tax rate continues to be τ in the future. We define the real "net debt" as the real debt minus the present value of future surpluses *under the existing tax policy*, $D_t(\tau) - V_t(\tau)$. The net debt measures the government's ability to repay its debt if it retains the existing tax policy.

We assume that the government faces an upper bound, ϕ, on the net debt–output ratio. The effective tax rate will remain at its "normal" level, τ_0, as long as the net debt-output ratio is below the upper bound. Whenever the ratio reaches or exceeds the upper bound, the tax rate is raised to a "crisis" level, τ^*, until the net debt, evaluated under the normal tax regime, falls to zero. At this point the effective tax rate is set back to its normal level. Thus, if the effective tax rate in period t is τ_t, then,

$$\tau_{t+1} = \begin{cases} \tau_0 & \text{if } \tau_t = \tau_0 \text{ and } D_t(\tau) - V_t(\tau_0) < \phi y_t \\ \tau_0 & \text{if } \tau_t > \tau_0 \text{ and } D_t(\tau) - V_t(\tau_0) \leq 0 \\ \tau^* & \text{if } \tau_t = \tau_0 \text{ and } D_t(\tau) - V_t(\tau_0) \geq \phi y_t \\ \tau^* & \text{if } \tau_t > \tau_0 \text{ and } D_t(\tau) - V_t(\tau_0) > 0 \end{cases} \tag{16}$$

where τ^* is set high enough so that the net debt declines on average over time: $D_t - V_t(\tau^*_{t+1}) < 0$. The level at which the crisis tax rate is set determines the average speed with which the net debt is reduced, and the normal tax regime is resumed. Given this policy rule, we can then evaluate the impact of risk management on welfare. Note that since τ_{t+1} is a function only of variables known at time t, its value can be predicted by agents one–period ahead.

"Maastricht" policy rule

The policy rule described by (16) is consistent with the empirical observations in Section 2, but is not uniquely so. An alternative policy rule might specify a change in tax regime whenever the debt (rather than the net debt) hits some bound. Indeed this is the kind of rule specified in the European Union's Stability Pact that is associated with the Maastricht Treaty.[14] Therefore, we also consider the implications of hedging under an alternative policy rule, given by

$$\tau_{t+1} = \begin{cases} \tau_0 & \text{if } \tau_t = \tau_0 \text{ and } D_t < \overline{\phi} y_t \\ \tau_0 & \text{if } \tau_t > \tau_0 \text{ and } D_t \leq \underline{\phi} y_t \\ \tau^* & \text{if } \tau_t = \tau_0 \text{ and } D_t \geq \overline{\phi} y_t \\ \tau^* & \text{if } \tau_t > \tau_0 \text{ and } D_t > \underline{\phi} y_t \end{cases} \tag{17}$$

There are several reasons why we prefer our benchmark policy rule. First, basing the rule on

[14] However, even the Stability Pact takes into account indicators of future prosperity other than current GNP, albeit indirectly.

the net debt ensures that the high tax regime is such that the government's intertemporal budget constraint is met.[15] Secondly, over the sample period the Canadian net debt rose monotonically, whereas the debt–GDP ratio of the early sixties exceeded the level reached in 1986. Finally, the Stability Pact rules are based on some kind of steady–state capacity to pay. In a long–run steady state, $V(\tau_0)$, would be a constant fraction of GDP, so that our benchmark rule would be equivalent to a Maastricht–type rule. Moreover, the Maastricht rules specify some margins such that the debt can violate the upper bound for short periods, which is similar in spirit to our use of a bound on the *net* debt–GDP ratio.

3.3 Hedging Strategy and Its Valuation

The Hedging Strategy

Without hedging, real government debt evolves according to the following equation:

$$D_t - D_{t-1} = r_{t-1}D_{t-1} - S_t = r_{t-1}D_{t-1} - [\tau_t - \overline{g} + \mathbf{a}'\mathbf{X}_t + \varepsilon_t]y_t. \tag{18}$$

For a given effective tax rate, the debt may increase rapidly if the surplus process experiences a large negative shock (e.g., a large negative value of $\mathbf{a}'\mathbf{X}_t$). *Ex ante*, the government can avoid some of these negative shocks through hedging. In this paper, we want to consider a hedging strategy that is potentially feasible to implement in practice. To do so, we require that the hedging to be done with nominal securities rather than real or inflation indexed securities. To avoid potential moral hazard problems, we further require that the US dollar value of the cash–flow from the hedging portfolio to be a fixed function of the market returns, $\mathbf{X}_t$, and unaffected by the domestic government's fiscal policy changes.

Let P_t be the US price level in period t and $\overline{Y}_t = e^{\mu_{\overline{y}} t}P_0 y_0$ denote the trend of domestic output measured in US dollar. So $\mu_{\overline{y}}$ is the average nominal output growth rate (in US dollars). Then, we can express the US dollar value of the primary surplus as

$$S_t^N = (\tau_t - \overline{g} + \varepsilon_t)P_t y_t + \mathbf{a}'\mathbf{X}_t(P_t y_t - \overline{Y}_t) + \mathbf{a}'\mathbf{X}_t\overline{Y}_t. \tag{19}$$

Here, we consider a simple hedging strategy that hedges the third component of the surplus process, $\mathbf{a}'\mathbf{X}_t\overline{Y}_t$. Specifically, the hedging strategy that we consider is the following: at some date t, the government starts to hold a portfolio whose payoff in US dollars in period $t+j$ is

$$h(\mathbf{X}_{t+j}, t+j) = \overline{s}\overline{Y}_{t+j} - (\mathbf{a}'\mathbf{X}_{t+j})\overline{Y}_{t+j}, \tag{20}$$

[15] In our simulations, the true present value of government surpluses, taking into account of potential tax changes, always exceeds the current debt.

where $\overline{s}$ is chosen so that the time t present value of the portfolio is zero. This hedging strategy effectively replaces a volatile component of the primary surplus with a deterministic cash–flow that is a constant percentage of the trend nominal output, and which has the same present value. One way to implement such a hedging strategy is by entering into an Index–Linked–Swap with investors, with the floating index being $(\mathbf{a}'\mathbf{X}_{t+j})\overline{Y}_{t+j}$.[16].

We focus our attention on such a hedging strategy for three reasons: First, variations in $\mathbf{a}'\mathbf{X}_t\overline{Y}_t$ account for about 60% of the variations in the Canadian primary surplus–output ratio. Therefore, hedging this component of the primary surplus process is likely to generate significant welfare gains. Second, since the value of $\mathbf{X}_t$ is determined in the US financial market independent of the Canadian government's actions, there are no transaction costs due to the potential for the government to partially default on its liability by inflating. Finally, as explained above, the hedging strategy is feasible in practice, because its cash-flow depends only on the returns of existing (nominal) assets.

Valuation

Under hedging, the real surplus process becomes

$$s^h_{t+j} = (\tau_{t+j} - \overline{g} + \varepsilon_{t+j} + \mathbf{a}'\mathbf{X}_{t+j})y_{t+j} + P^{-1}_{t+j}(\overline{s} - \mathbf{a}'\mathbf{X}_{t+j})\overline{Y}_{t+j} \tag{21}$$

and the real present value of the primary surpluses is

$$V^h_t(\tau) = V_t(\tau) + \frac{1}{M_t}\sum_{j=1}^{\infty} E_t M^n_{t+j}(\overline{s} - \mathbf{a}'\mathbf{X}_{t+j})\overline{Y}_{t+j}. \tag{22}$$

where

$$M^n_{t+j} = M_{t+j}P^{-1}_{t+j} \tag{23}$$

is the nominal stochastic discount factor for pricing securities denominated in US dollars.

By diversifying the market risk, the debt process under hedging is less volatile. However, whether the net debt is less likely to hit the upper bound under hedging depends on the value of $\overline{s}$, which is determined by

$$\sum_{j=0}^{\infty} E_t[M^n_{t+j}\overline{s}\overline{Y}_{t+j}] = \sum_{j=0}^{\infty} E_t[M^n_{t+j}\mathbf{a}'\mathbf{X}_{\mathbf{t+j}}\overline{Y}_{t+j}], \tag{24}$$

or

$$\overline{s} = \frac{\sum_{j=0}^{\infty} E_t[M^n_{t+j}\mathbf{a}'\mathbf{X}_{t+j}\overline{Y}_{t+j}]}{\sum_{j=0}^{\infty} E_t[M^n_{t+j}\overline{Y}_{t+j}]}. \tag{25}$$

Since the vector $\mathbf{X}_{t+j}$ is demeaned, its expected value equals zero. So, we can rewrite equation (25)

[16] Various kinds of Index–Linked–Swaps have now been widely traded by many financial institutions, although not with infinite maturity. We discuss the potential problems of implementing such a swap in the conclusion.

as

$$\overline{s} = \frac{\sum_{j=0}^{\infty} \text{Cov}_t[M^n_{t+j}, \mathbf{a}'\mathbf{X}_{t+j}]\overline{Y}_{t+j}}{\sum_{j=0}^{\infty} E_t[M^n_{t+j}\overline{Y}_{t+j}]}, \tag{26}$$

which shows clearly that $\overline{s}$ reflects the financial cost of hedging — the risk premium that must be paid to investors. If the risk associated with $\mathbf{a}'\mathbf{X}_{t+j}\overline{Y}_{t+j}$ is fully diversifiable, i.e., the covariance between M^n_{t+j} and $\mathbf{a}'\mathbf{X}_{t+j}$ is zero for all $j > 0$, then $\overline{s} = 0$. When $\text{Cov}_t[M^n_{t+j}, \mathbf{a}'\mathbf{X}_{t+j}] <0$ for all $j > 0$, then $(\mathbf{a}'\mathbf{X}_{t+j})\overline{Y}_{t+j}$ contains a component of systematic risk and the government has to pay a risk–premium for downloading the risk to investors. In this case, the cash–flow the government receives, $\overline{s}\overline{Y}_{t+j}$, is actually negative. The debt will grow faster on average under hedging if the size of this negative cash–flow is too large.

3.4 Modeling the Shocks and Asset Pricing

In order to quantify the hedging cost and to evaluate domestic households' welfare, we need to specify a joint stochastic process for the shock variables $\mathbf{X_t}$, the discount factors M_t and M^n_t, the US price level P_t, the effective real interest rate on domestic government debt r_t, and domestic productivity ψ_t.

The Process for $\mathbf{X_t}$

We assume that the vector $\tilde{\mathbf{X}}_t = (X_{1,t}, X_{2,t}, X_{3,t}, X_{4,t})$, which consists of the demeaned asset returns VWR, DIV, LONGR and TBILL, follows a vector autoregressive (VAR) process:

$$\tilde{\mathbf{X}}_t = \mathbf{A}\tilde{\mathbf{X}}_{t-1} + \mathbf{u}_t \tag{27}$$

where $\mathbf{A}$ is a matrix of coefficients, $\mathbf{u}_t$ is i.i.d., and $\mathbf{u}_t \sim N(\mathbf{0}, \mathbf{\Sigma})$. Note that the vector $\tilde{\mathbf{X}}_t$ does not include one of the factors we used in $\mathbf{X}_t$ —TBMA, the one–year moving average of TBILL. However, given the estimated process for the asset returns, the value of TBMA can be easily constructed. The process is estimated using quarterly data from 1958:1 to 1994:4.

The Stochastic Discount Factors

Since we use the stochastic discount factors to value cash-flows that are functions of $\tilde{\mathbf{X}}_t$, it is important to model the covariance between the stochastic discount factors and $\tilde{\mathbf{X}}_t$. Here, we adopt the following linear specification for the growth rate of the nominal stochastic discount factor:

$$-\ln\left(\frac{M^n_t}{M^n_{t-1}}\right) = \mu_n + \mathbf{b}'_n\tilde{\mathbf{X}}_{t-1} + \omega_{n,t}, \tag{28}$$

where $\omega_{n,t}$ is i.i.d., $\omega_{n,t} \sim N(0, \sigma^2_n)$, and $E\left[\omega_{n,t}\mathbf{u}_t\right] = \mathbf{v}_n$. We further assume that the expected US inflation rate is also a linear function of $\tilde{\mathbf{X}}_{t-1}$:

$$\ln\left(\frac{P_t}{P_{t-1}}\right) = \mu_p + \mathbf{b}_p'\tilde{\mathbf{X}}_{t-1} + \omega_{p,t}, \tag{29}$$

where $\omega_{p,t}$ is i.i.d., $\omega_{p,t} \sim N(0, \sigma_p^2)$, $E\left[\omega_{p,t}\mathbf{u}_t\right] = \mathbf{v}_p$ and $E\left[\omega_{p,t}\omega_{n,t}\right] = \sigma_{pn}$. It follows that the real stochastic discount factor is given by

$$-\ln\left(\frac{M_t}{M_{t-1}}\right) = \mu_m + \mathbf{b}_m'\tilde{\mathbf{X}}_{t-1} + \omega_{m,t}, \tag{30}$$

where $\mu_m = \mu_n - \mu_p$, $\mathbf{b}_m = \mathbf{b}_n - \mathbf{b}_p$, and $\omega_{m,t} = \omega_{n,t} - \omega_{p,t}$.

Model 1: In our benchmark calculations, we focus on a special case of this specification. We extend the term structure model discussed in Campbell and Viceira (1998) and Campbell, Lo and MacKinlay (1997) by allowing the innovation in the nominal stochastic discount factor to be correlated with the innovations in the shock variables.

$$-\ln\left(\frac{M_t^n}{M_{t-1}^n}\right) = r_{t-1}^n + \frac{1}{2}\sigma_n^2 + \omega_{n,t}, \quad \omega_{n,t} \sim N(0, \sigma_n^2), \tag{31}$$

where r_{t-1}^n is the 3-month risk–free nominal interest rate, and $\omega_{n,t}$ is correlated with $\mathbf{u}_t$. By definition, $X_{4,t-1} = 4(r_{t-1}^n - E\left[r_t^n\right])$. So, the above equation is a special case of (28) with $\mu_n = E\left[r_{t-1}^n\right] + \frac{1}{2}\sigma_n^2$, and $\mathbf{b}_n' = (0, 0, 0, 1/4)$.

Model 2: We also report results using the Epstein–Zin consumption-based CAPM. The real discount factor implied by this model is

$$M_t = \left(\beta^t C_t^{-\frac{1}{\sigma}}\right)^{\theta} \left(\Pi_{s=0}^t \frac{1}{R_s^m}\right)^{1-\theta}. \tag{32}$$

where C_τ denotes real per capita consumption, R_t^m denotes the gross real return on the market portfolio, σ is the elasticity of intertemporal substitution, and $\theta = (1-\gamma)/1 - (1/\sigma)$, where γ is the coefficient of relative risk aversion.[17]

Domestic Productivity

We assume that domestic productivity evolves according to:

$$\psi_t = \psi_0 e^{\mu_\psi t + \mathbf{b}_\psi'\tilde{\mathbf{X}}_{t-1} + \nu_t}, \tag{33}$$

where $\nu_t \sim N(0, \sigma_\nu^2)$ is an i.i.d. process independent of $\omega_{m,t}$, the innovation in the growth rate of the real stochastic discount factor, and μ_ψ is the expected productivity growth rate. Note that we

[17] Appendix B details how we implemented this model.

allow for potential correlations between total factor productivity movements and the lagged shock variables $\tilde{\mathbf{X}}_{t-1}$. These correlations reflect the international components of domestic productivity shocks. From (8) and the fact that the tax rate is known one period in advance, real output is given by

$$y_t = \psi_t \left(\frac{\alpha(1-\tau_t)E_{t-1}\left[\frac{M_t}{M_{t-1}}\psi_t\right]}{1-(1-\delta(1-\tau_t))E_{t-1}\left[\frac{M_t}{M_{t-1}}\right]} \right)^{\frac{\alpha}{1-\alpha}} \tag{34}$$

The Effective Real Interest Rate on Government Debt

The effective real interest rate on domestic government debt r_t is also assumed to depend on lagged shock variables:

$$r_t = c_r + \mathbf{b}_\mathbf{r}'\tilde{\mathbf{X}}_{t-1} + \rho_r r_{t-1} + \omega_{r,t}, \tag{35}$$

where $\omega_{r,t}$ is a zero mean, i.i.d. normal variable. The effective real interest rate is measured as $(1+r^D_{t-1})/\pi_t$, where r^D_{t-1} is the ratio of interest payment on debt to the stock of domestic government debt and π_t is the *ex post* domestic inflation rate between period $t-1$ and t.

3.5 Benchmark Calibration

Asset Pricing Model: Let r^n_t be the interest rate on the 3-month Treasury bills, $r^{n,L}_t$ be the yield on 10-year Treasury bonds, $R^{n,m}_t$ be the nominal return on market portfolio. Then, the following no–arbitrage conditions should hold for an asset pricing model:

$$E_t\left[\frac{M^n_{t+1}}{M^n_t}\exp(r^n_t)\right] = 1, \tag{36}$$

$$E_t\left[\frac{\frac{1}{2}\left(\sum_{j=1}^{20} M^n_{t+2j} r^{n,L}_t\right) + M^n_{t+40}}{M^n_t}\right] = 1, \tag{37}$$

$$E_t\left[\frac{M^n_{t+1}}{M^n_t} R^{n,m}_{t+1}\right] = 1. \tag{38}$$

where $r^n_t = E\left[r^n_t\right] + \frac{1}{4}X_{4t}$, $r^{n,L}_t = E\left[r^{n,L}_t\right] + X_{3t}$ and $R^{n,m}_t = \exp(X_{1t})$.

For the asset pricing models that we consider, we calibrate the parameters so that these three conditions hold on average over the sample period 1977:1–1994:4.[18] For the term structure model (31), the first moment condition (36) is always satisfied by construction. We further assume that

$$\omega_{n,t} = \rho_1 u_{1,t} + \rho_4 u_{4,t}. \tag{39}$$

[18] The asset pricing models we consider are homoskedastic, which implies that both the term premium and the equity premium are constant over time. In the data, however, these premiums are time–varying. Therefore, the calibrated parameters will be dependent on the sample period. Since we are concerned about values at the beginning of 1977, we choose 1977:1-1994:4 as the sample period for the calibration exercise.

That is, the innovation in the nominal stochastic discount factor is a linear combination of the innovation in the 3-month interest rate and the innovation in the return on market portfolio. Then, we have

$$\sigma_n^2 = \rho_1^2 \sigma_{1,u}^2 + \rho_4^2 \sigma_{4,u}^2 + 2\rho_1 \rho_4 \sigma_{14,u}, \tag{40}$$

$$\mathbf{v}_n' = (\rho_1, 0, 0, \rho_4)' \boldsymbol{\Sigma}. \tag{41}$$

We use the other two moment conditions (37)–(38) to calibrate the parameters ρ_1 and ρ_4. For more details on calibration, see Appendix A.

Output: In accordance with much of the real business cycle literature, we set $\alpha = 0.34$, $\delta = 0.02$. We calibrate the productivity growth process so that the implied stochastic process followed by the output growth generated by the model matches that in the Canadian data. The details of the calibration are given in Appendix A.

Fiscal Policy Rule: For the policy rule to be fully specified, we must choose values for τ_0, τ^* and ϕ. The regression analysis in section 2 implies that

$$\tau_t = \begin{cases} \tau_0 = 0.1737, & \text{if } t \le t^*; \\ \tau^* = 0.1930, & \text{if } t > t^*. \end{cases} \tag{42}$$

We interpret t^* as the time when the net debt–GDP ratio hit the boundary ϕ. Thus, under the normal tax regime, the primary surplus process evolves according to the following equation

$$s_t = \tau_0 - \overline{g} + \mathbf{a}' X_t + \varepsilon_t, \tag{43}$$

where the values of $\mathbf{a}$ is determined from the regression analysis in section 2. From the asset pricing model specified above we can determine the present value $V_{t^*}(\tau_0)$, and the value of ϕ is then set to be the net debt–GDP ratio at t^*, $(D_{t^*} - V_{t^*}(\tau_0))/y_{t^*}$. Given our estimate that the shift in policy stance occurred after the first quarter of 1986, the implied value of the upper bound on the net debt/GDP ratio, ϕ, is about 0.05.[19] We use this value for our benchmark economy, but consider the sensitivity of our results to alternative values.

Initial Conditions: We take the perspective of a government that starts to hedge in 1977:1 and has only the information available up to the last quarter of 1976. The sixth column of Table 1 (titled "pre–77") documents the results of estimating the model over the period 1958:1 to 1976:4. As can be seen the coefficient estimates are quite robust to this truncation of the sample period. The fact that we are able to identify the replicating portfolio *ex ante* suggests that our empirical specification should provide a useful basis for hedging the shocks to the surplus. Indeed, as Figure 3 illustrates,

[19] Note that debt is a stock variable while GDP is a flow variable. We express debt–GDP ratios as annualized figures, following the convention in the policy literature.

a forecast conditional on the realized asset returns and the shift in policy stance performs rather well in replicating the actual surplus in the post–sample period 1977:1 to 1994:4. This conditional forecast replicates over 80% of the variation in the surplus.

— FIGURE 3 —

3.6 Results

Using the data from our simulation, we estimate the probability that the initial policy first becomes unsustainable and the government is forced to raise the tax rate. To do this we counted the fraction of paths along which the tax is increased for the first time. Figure 4 shows the evolution of this probability after $t =$1977:1 both with and without hedging. Without hedging, the probability that the original policy rule would have become unsustainable within the following 200 quarters (i.e. by the last quarter of 2026), is $Q(200) = 0.26$. With hedging it drops to $Q_H(200) = 0.05$. But what does this five–fold increase in sustainability imply for the expected tax rate and welfare ?

— FIGURE 4 —

Figure 5 shows the average tax rates that result from the policy rule in the benchmark economy, with and without hedging over 500 quarters. As can be seen, the expected tax rates rise initially in both cases. The average tax rate rises less rapidly with hedging than without, reflecting the reduced likelihood of hitting the upper bound on the net debt level. Once the tax has been increased, the net debt begins to decline on average so that eventually, the tax rate can be reduced to its "normal level". Over time there are more paths realizing falling taxes on average than there are paths realizing rising taxes, so that the average tax rate falls.[20] In the long run, the tax rate remains lower under hedging because (1) the probability of having to increase taxes in the future is lower, and (2) the average rate at which the net debt is reduced once the tax is raised is greater.

— FIGURE 5 —

We compute the welfare gain arising from hedging using (9). The first row of Table 2 shows the aggregate wealth levels relative to current GDP with and without hedging, and the welfare change for the benchmark economy. In particular it shows that, although the expected financial cost of

[20] Note that even after the net debt is reduced to zero, some paths realize sufficiently bad shocks to make the net debt positive again and to eventually experience rising taxes again. However, the average tax rate still declines.

hedging is 0.14% of GDP, the percentage increase in production wealth is 0.36%. In principle, this welfare gain could come from two sources: (1) the reduction in expected taxes and (2) the reduced variation in taxes via the concavity of the production function. However, by far the greatest part of the gain comes from the former. Indeed we find that over 90% of the welfare gain derives from the reduction in average taxes.[21] In other words, in our model, the gains from hedging come predominantly from the increased sustainability of the low tax policy.

— TABLE 2 —

To investigate the nature of these welfare gains further and to assess the sensitivity of our results to the various assumptions we have made, we allow the key underlying parameters to vary from our benchmark case. The results are given in Table 2 and are discussed below.

The Hedgable Component of the Surplus: The hedging strategy we have adopted does not allow us to create new assets. This means we have ruled out the possibility of hedging a significant part of the stochastic component of the surplus — that part equal to the product of the asset returns and the deviations of output from trend. The second row of column 2 shows the results for the benchmark case if we were to hedge the entire stochastic component of the surplus, $X_t Y_t$. As can be seen the percentage welfare gain increases substantially to 0.63%. Note however that the transactions costs associated with introducing such assets are unknown, so that this should be viewed as an upper bound on the possible welfare gains in the benchmark economy.

The Equity Premium: We re–calibrated the asset pricing model to obtain new values of ρ_1 and ρ_4, that were consistent with different values of the equity premium, while at the same time maintaining the risk–free rate and the term premium. Results for an equity premium that is 1% lower and 1% higher than in the benchmark case are shown in Table 2. As one might expect, lowering (raising) the equity premium increases (decreases) welfare whether the government hedges or not. Moreover, the welfare *gain* from hedging declines with the equity premium. There are two main effects from raising the equity premium:

• The risk premium required by the representative investor increases. This drives up the expected financial cost of hedging, $-\overline{s}$, which feeds into higher expected taxes under hedging, thereby reducing the welfare gain.

[21] This is generally the case under all of the alternatives considered below.

• The present value of future surpluses under the initial policy, $V(\tau_0)$, decreases. The resulting higher initial net debt reduces the sustainability of the initial tax policy and may increase or reduce the welfare gain.

To separate the effects of changing the risk premium from that of changing initial conditions, we also report in Table 2 results for the low and high equity premia when the debt level is adjusted so that the initial net debt is the same as that in the benchmark case. In the low risk case the adjustment involves raising the initial debt. In the high risk case it involves lowering the initial debt. After controlling for the initial net debt in this way, the welfare gain *decreases* in both cases. This reflects the non–monotonic relationship between the initial net debt and the welfare gain (see below).

Hedging Cost $(-\overline{s})$**:** To isolate the role of the increase in the financial cost of hedging from other aspects of the increase in risk aversion, we fixed the values of ρ_1 and ρ_4, but varied the hedging cost, $-\overline{s}$ directly, so that it would equal the values associated with 1% lower or higher values of the equity premium. As can be seen, raising this hedging cost lowers the welfare gain and lowering it raises the welfare gain. However, the difference is not very large implying that most of the change in the welfare gain is coming from the implied change in the level of initial net debt.

Initial Debt (D_0/y_0)**:** In the benchmark case, the initial debt–GDP ratio is $D_0/y_0 = 0.20$. Raising it lowers welfare because the increase makes the need for a tax hike more imminent (as indicated by the high probability of a tax hike within the first 200 quarters, $Q(200)$). The effect of the initial debt on the percentage welfare gain from hedging, however, is non–monotonic. When the initial debt is far below the upper bound, the probability of the net debt hitting the upper bound is very small and, therefore, the welfare gain from hedging is small. As the initial debt increases, the probability of the net debt hitting the upper bound increases and the welfare gain from hedging increases as well. If the initial net debt is too close to the upper bound, however, hedging actually *increases* the probability of a tax hike. With a high initial level and a strictly positive effective interest rate the net debt is expected to rise and hit the upper bound quickly, and hedging reduces the chance of having positive shocks to the surplus that would help to revert the upward trend. This reduction of the sustainability reduces welfare gain. Interestingly, however, there are still welfare gains from hedging, reflecting the more rapid reduction in the net debt under hedging following the first tax increase and the lower likelihood of further tax hikes in the future.

The non–monotonicity in the relationship between the welfare gain and the initial debt explains our earlier results associated with the adjustment of the initial debt to offset the effects of changing the risk–premium.

Starting Date (t_0): The starting date for the adoption of the hedging strategy in the benchmark economy, t_0 =1977:1, was approximately in the middle of the sample. This is a date at which the debt level and the realized values of the asset returns are such that the initial net debt is well below the trigger point and the hedging cost takes on a relatively low value. We also ran our simulation with a starting date of t_0 =1986:1. As noted above this is a date at which the net debt is just about to exceed the upper bound and it is also a date such that the implied cost of hedging is still quite high. The combination of these two factors reduces the welfare gain from hedging significantly. Note, however, that the welfare gains are still positive because of the more rapid reduction in the net debt under hedging following the first tax increase, and the lower likelihood of further tax hikes in the future.

Upper Bound on the Net Debt (ϕ)**:** Lowering the net debt–GDP ratio at which the government increases taxes, ϕ, to 0.025 raises the likelihood of tax increases, thereby raising the expected tax rate and lowering welfare. Raising ϕ to 0.075 has the opposite effects. However, the welfare *gain* increases in both cases. This non–monotonicity arises for similar reasons to those described for changes in the initial net debt–GDP ratio.

"Maastricht" Policy Rule: We also include results based on a policy rule in which tax changes occur when the debt–GDP ratio reaches lower and upper bounds. For the upper bound we use the value that the debt–GDP ratio reached in 1986:2 (approximately 0.50), and we somewhat arbitrarily set the lower bound to equal 0.125.[22] The results are shown in Table 2 for the benchmark parameters. As can be seen welfare levels are very similar for this policy rule, however, the welfare gain is somewhat greater. Since the lower bound of 0.125 implies that the tax rate may be set at a high level for an unreasonably long period of time, we also show the results when the lower critical value is 0.25. The welfare levels are a little greater than with the lower minimum level, but the percentage welfare gain is smaller. These results reflect the fact that the high tax regime is less costly because it does not last as long on average.

A Consumption–Based Asset Pricing Model: We have based our analysis on a two–factor asset pricing model. The advantage of this model is that it is possible to calibrate it to match the key moments in the data (i.e. the risk–free rate, the equity premium and the term premium). For the sake of comparison, we also include results using the Epstein–Zin consumption–based asset pricing model. In principal, one could use all three moment conditions (36)–(38) to calibrate the parameters

[22] Although under the benchmark policy rule, sustainability requires that the net debt reach zero before taxes are reduced, imposing the same requirement on the debt seems extreme.

β, γ and σ.[23] Unfortunately, it turns out that it is not possible to do this while also ensuring that the forward rate is large enough for present values to converge (see Appendix B). Instead, we use parameter values that are consistent with those typically assumed in the literature and which yield the same forward rate as in the benchmark case. The last two rows of Table 2 give the results for two cases: a low risk premium case ($\beta = 0.99$, $\gamma = 4$ and $\sigma = 0.5$) and a high risk premium case ($\beta = 0.99$, $\gamma = 10$, and $\sigma = 0.321$). As can be seen the hedging cost is lower in both cases and the welfare *gain* is somewhat higher than in the benchmark case.

4. Concluding Remarks

The central premise of this paper is that government cash flows are subject to unavoidable fiscal shocks that are outside the control of the fiscal authorities. In this paper we replicate many of the shocks to the Canadian federal surplus using the return on a linear combination of U.S. asset returns. We find that it is possible to characterize the surplus process over the last four decades as a stationary function of these shocks with an abrupt regime shift in 1986. Our results are consistent with the hypothesis that the recent rise in public debt experienced by Canada was the result of a series of negative shocks in the 1970s and 1980s, and a long delay in the adjustment of fiscal policy in response.

Although some fiscal shocks could be offset by varying tax rates and other policy parameters, this would create further distortions in the economy. The alternative of intertemporal smoothing through debt financing is ultimately unsustainable. We have argued in this paper that, because of this conflict between stability and sustainability, systematic fiscal risk management might be beneficial as part of the government's overall debt management strategy. We explored the feasibility of this, and estimated the potential gains from fiscal risk management in terms of increased sustainability, reduced tax rates and welfare. The increases in sustainability are large, and there are non–trivial welfare gains (on the order of one half of one percent). It should be noted that some of our modeling assumptions are somewhat conservative. In particular, if we allowed for endogenous growth, then tax changes would have permanent effects, so that the welfare gains from hedging would be much larger.

In this paper, we have abstracted from several interesting and potentially important issues regarding the implementation of a fiscal risk management strategy. The hedging strategy that we considered requires the government to enter into an index–linked swap with an infinite maturity.

[23] We illustrate how this can be done in Appendix B.

It would be interesting to see if the strategy can be replicated with more conventional financial instruments. There is also the issue of default risk that is often associated with swaps of long maturity. In this paper we have dealt with this problem to some extent by having the payoffs of the swap denominated in US dollars. This eliminates the possibility of partial default by the Canadian government through inflation. Of course, this does not exclude the possibility of direct or indirect default by the government through other means, and it would be interesting to evaluate the welfare gains from hedging by taking into account credit risk explicitly.[24] Finally, there is the issue of time–consistency. In this paper we assume that once the government decides to implement the hedging strategy, it will stick to it in the future. However, our simulations show that the gains from hedging depend crucially on the initial level of the net debt. A hedging strategy that is welfare improving *ex ante* may become welfare reducing *ex post* if a series of adverse and unhedgable shocks occur that cause the net debt to increase significantly in the future. Finding a welfare–enhancing hedging strategy that is also time–consistent is another interesting avenue for further research.

We have used Canadian federal finances to illustrate the importance of exogenous fiscal shocks to the rising public debt problems and to explore the feasibility and desirability of systematic fiscal risk management. However, the conceptual framework and empirical methodology that we have employed here could also be applied to the fiscal problems of other OECD countries (especially those in the EMU), as well as to those of the US states and Canadian provinces. Such analysis is important given the emphasis on governments' responses to fiscal shocks in the current literature on budget deficits.

In analyzing the role of fiscal risk management, we have focused on diversifiable shocks to the government's primary surplus. Since not all the shocks to the primary surplus can be hedged away, there is still a need for the government to smooth cash flows intertemporally by issuing risk–free bonds. Thus, debt management in the form of hedging interest rate risk and choosing the optimal maturity structure as suggested by, among others, Boothe and Reid (1991), Missale and Blanchard (1994) and Barro (1997) are also important for maintaining stability and enhancing the sustainability of fiscal policy. The risk management strategy we emphasize here is complementary to their suggestions on debt management.

[24] Note, however, that hedging should reduce the default risk premium already implicit in the effective interest rate on the debt, thereby offsetting the increased cost of hedging.

Appendix

Data

All of the data and the code used in this paper can be downloaded from the internet at http://www.chass.utoronto.ca/~lloydell/papers/fiscrisk.html. A not–for–publication appendix, referred to as Appendix B is also available at this location.

Fiscal Variables

The quarterly primary surplus was calculated as the difference between total federal revenues and expenditures less interest payments on the debt, as published by Statistics Canada. For institutional reasons, this data exhibits considerable seasonal variation. Specifically, annual crown corporation cash flows are attributed only to the second quarter yielding a large "spike". We therefore used seasonally adjusted data. The surplus data does not include charges and subsidies relating to the Petroleum Compensation fund. Quarterly public debt figures are taken from IMF International Financial Statistics. The effective interest rate was calculated as the ratio of actual interest payments on the debt to value of the debt.

Asset Returns

VWR is the index of value–weighted returns on the NYSE taken from the CRSP tape. DIV is the dividend yield on the NYSE from the CRSP tape. LONGR is the nominal interest rate on 10 year US. government bonds. TBILL is the nominal 3–month US. treasury bill rate. TBMA is a one–year fixed–weight moving average of TBILL. All of these returns were converted into Canadian dollars using the spot U.S.–Canadian exchange rate taken from CITIBASE. Note that these returns should therefore be interpreted as the return in Canadian dollars on each U.S. dollar invested.

Data used to Compute the Epstein–Zin Stochastic Discount Factor

Real per capita US consumption was calculated using data from CITIBASE. The real rate of return on the market portfolio was taken to be equal to VWR divided by the US CPI.

Details of Calibration

Calibration of the Real Output Process

Given our specifications for the real stochastic discount factor and productivity, we have

$$y_t = \left(\psi_0 e^{\mu_\psi t + \mathbf{b}'_\psi \tilde{\mathbf{X}}_{t-1} + \nu_t}\right) \left(\frac{\alpha(1-\tau)e^{\mu_\psi t + \mathbf{b}'_\psi \tilde{\mathbf{X}}_{t-1} + \frac{1}{2}\sigma_\nu^2 - \mu_m - \mathbf{b}'_m \tilde{\mathbf{X}}_{t-1} + \frac{1}{2}\sigma_m^2}}{1-(1-\delta(1-\tau))e^{-\mu_m - \mathbf{b}'_m \tilde{\mathbf{X}}_{t-1} + \frac{1}{2}\sigma_m^2}}\right)^{\frac{\alpha}{1-\alpha}}. \tag{A1}$$

Taking logs on both sides yields

$$\ln y_t = \frac{1}{1-\alpha}\left(\ln\psi_0 + \frac{1}{2}\alpha\sigma_v^2 + \alpha\ln\left(\alpha(1-\tau)\right)\right) + \frac{\mu_\psi}{1-\alpha}t + \frac{\mathbf{b}_\psi'\tilde{\mathbf{X}}_{t-1}}{1-\alpha} + \nu_t \\ -\frac{\alpha}{1-\alpha}\ln\left(e^{\mu_m - \frac{1}{2}\sigma_m^2 + \mathbf{b}_m'\tilde{\mathbf{X}}_{t-1}} - (1-\delta(1-\tau))\right) \quad \text{(A2)}$$

Now taking a first–order linear approximation around $\mathbf{b}_m'\tilde{\mathbf{X}}_{t-1} = 0$ for the last term, we can write

$$\ln y_t \approx q_0(\tau) + \mu_y t + \mathbf{q}(\tau)'\tilde{\mathbf{X}}_{t-1} + \nu_t \quad \text{(A3)}$$

where

$$q_0(\tau) = \frac{1}{1-\alpha}\left(\ln\psi_0 + \frac{1}{2}\alpha\sigma_v^2 + \alpha\ln\left(\alpha(1-\tau)\right)\right) - \frac{\alpha}{1-\alpha}\ln\left(e^{\mu_m - \frac{1}{2}\sigma_m^2} - (1-\delta(1-\tau))\right), \quad \text{(A4)}$$

$$\mathbf{q}(\tau) = \frac{1}{1-\alpha}\left(\mathbf{b}_\psi - \frac{\alpha e^{\mu_m - \frac{1}{2}\sigma_m^2}}{e^{\mu_m - \frac{1}{2}\sigma_m^2} - (1-\delta(1-\tau))}\mathbf{b}_m\right), \quad \text{(A5)}$$

and

$$\mu_y = \overline{\mu}_y - \mu_p - \frac{1}{2}\sigma_p^2. \quad \text{(A6)}$$

Regressing $\ln y_t - \mu_y t$ on the state variables over a period during which the effective tax rate is deemed constant yields the coefficients q_0, and $\mathbf{q}$, plus the variance of ν_t, σ_ν^2. The stochastic process followed by the productivity parameter can then be backed out using the method of undetermined coefficients.[25]

Calibrating the Two–factor Asset Pricing Model

From (31), (39) and (40), the moment condition (38) can be written as follows:

$$\exp\left(E_t\left[R_{t+1}^{m,n}\right] - r_t^n + \frac{1}{2}\sigma_{1,u}^2 - \rho_1\sigma_{1,u}^2 - \rho_4\sigma_{14,u}\right) = 1 \quad \text{(A7)}$$

or

$$E_t\left[R_{t+1}^{m,n}\right] - r_t^n + \frac{1}{2}\sigma_{1,u}^2 - \rho_1\sigma_{1,u}^2 - \rho_4\sigma_{14,u} = 0. \quad \text{(A8)}$$

Taking unconditional expectations of the left side of the equation yields

$$E\left[R_t^{m,n} - r_t^n\right] + \frac{1}{2}\sigma_{1,u}^2 - \rho_1\sigma_{1,u}^2 - \rho_4\sigma_{14,u} = 0. \quad \text{(A9)}$$

Replacing the theoretical moments with sample moments, we have

$$\frac{1}{T}\sum_{t=1}^{T}\left(R_t^{m,n} - r_t^n\right) + \frac{1}{2}\sigma_{1,u}^2 - \rho_1\sigma_{1,u}^2 - \rho_4\sigma_{14,u} = 0. \quad \text{(A10)}$$

[25] Note also that $\mu_\psi = (1-\alpha)\mu_y$.

Using (28) and (31) we have

$$E_t\left[\frac{M^n_{t+2j}}{M^n_t}\right] = \exp\left(-\left(r^n_t + \frac{1}{2}\sigma^2_n\right)j - m^n_z(t,2j) + \frac{1}{2}V^n_{zz}(t,2j)\right). \tag{A11}$$

where $m^n_z(t,2j) = E_t\left[-\ln M^n_{t+2j} - 2\mu_n j\right]$ and $V^n_{zz}(t,2j) = E_t\left[\left(-\ln M^n_{t+2j} - 2\mu_n j - m_z(t,2j)\right)^2\right]$. So, the moment condition (37) can be written as

$$\begin{aligned} 1 &= \frac{1}{2}\left[\sum_{j=1}^{20}\exp\left(-\left(r^n_t + \frac{1}{2}\sigma^2_n\right)j - m^n_z(t,2j) + \frac{1}{2}V^n_{zz}(t,2j)\right)\right] r^{n,L}_t \qquad \text{(A12)} \\ &\quad + \exp\left(-\left(r^n_t + \frac{1}{2}\sigma^2_n\right)40 - m^n_z(t,40) + \frac{1}{2}V^n_{zz}(t,40)\right). \end{aligned}$$

Taking the sample average of the RHS of this equation yields

$$\begin{aligned} 1 &= \frac{1}{T}\sum_{t=1}^{T}\left\{\frac{1}{2}\left[\sum_{j=1}^{20}\exp\left(-\left(r^n_t + \frac{1}{2}\sigma^2_n\right)j - m^n_z(t,2j) + \frac{1}{2}V^n_{zz}(t,2j)\right)\right] r^{n,L}_t\right\} \\ &\quad + \frac{1}{T}\sum_{t=1}^{T}\exp\left(-\left(r^n_t + \frac{1}{2}\sigma^2_n\right)40 - m^n_z(t,40) + \frac{1}{2}V^n_{zz}(t,40)\right). \qquad \text{(A13)} \end{aligned}$$

We choose the values of ρ_1 and ρ_4 so that they are the solutions to the equations (A10) and (A13). We do so by first using (A10) to express ρ_1 as a linear function of ρ_4 and substituting it into equation (A13). We then numerically look for the value of ρ_4 that solves equation (A13).

Computation of Fiscal Policy, Sustainability and Welfare

To compute the probability of policy shifts and the associated welfare impacts, we conducted a Monte Carlo simulation. For each set of parameters, we estimated the underlying VAR to determine the parameters of the system and the associated joint distribution of the errors. We used this to generate N paths of T periods for the entire system. For each path and at each date, we computed the implied debt level, D_t, and the present value of future forecasted primary surpluses under the current policy, $V_t(\tau_t)$. We did this for both the hedged and unhedged government cash flow processes. We then computed the associated net debt and used it to determine the tax rate to be set in the next period according to the policy rule described in (16). This generated a joint numerical distribution over the tax rate and the discount factor which we used to compute welfare. Since changes in the tax rate occur infrequently (i.e. only when the bounds on the net debt are hit), a large number of paths and time periods were required before our estimated welfare gain converged. Specifically, $N = 5,000,000$ and $T = 500$ were sufficient for convergence of the welfare gain estimate up to the second decimal place.

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Table 1 – Decomposing the Primary Surplus

	Variable	Shock	Lag	Debt	Dummy	Shift	Pre–77	No Lag
X	VWR	0.005	0.012	0.006	0.003	0.003	0.003	-0.004
		(0.57)	(1.56)	(0.90)	(0.43)	(0.43)	(0.04)	(0.55)
	DIV	-1.28	-0.54	-0.64	-0.69	-0.69	-0.70	-1.09
		(12.97)	(5.01)	(6.06)	(7.04)	(7.09)	(5.05)	(14.18)
	LONGR	-0.22	-0.08	-0.22	-0.33	-0.33	-0.30	-0.51
		(3.57)	(1.59)	(3.73)	(5.62)	(5.68)	(2.53)	(9.24)
	TBILL	0.19	0.14	0.22	0.27	0.27	0.22	0.35
		(3.34)	(3.29)	(4.88)	(6.26)	(6.34)	(2.47)	(7.71)
	TBMA	0.38	0.09	0.22	0.22	0.22	0.28	0.41
		(5.92)	(1.46)	(3.13)	(3.82)	(3.97)	(2.47)	(8.46)
	Constant	0.0534	0.0218	0.0167	0.0319	0.0317	0.0319	0.0506
		(18.70)	(5.48)	(4.21)	(6.64)	(8.39)	(5.35)	(23.15)
	SLAG	—	0.61	0.50	0.38	0.38	0.24	—
			(9.58)	(7.77)	(5.83)	(5.86)	(2.24)	
	DEBT	—	—	0.0070	-0.0002	—	—	—
				(4.18)	(0.09)			
	DUM	—	—	—	0.0109	0.0108	—	0.0157
					(4.91)	(6.70)		(10.30)
	NOBS	148	148	148	148	148	76	148
	$\overline{R}^2$	0.68	0.81	0.83	0.85	0.85	0.75	0.82
	D–W	0.69	2.06	2.06	2.03	2.04	1.96	1.20
	CUSUM	5.69	3.63	3.21	—	—	—	—
	FTEST	25.6	7.30	5.00	—	—	2.09	—
		[.000]	[.000]	[.000]	—	—	[0.08]	—

Notes:

(1) t–statistics are given in parenthesis.

(2) P–values in square brackets.

(3) In the first 3 columns, FTEST refers to a Chow test for a structural break in 1986:2. In the sixth it refers to a test of whether the coefficients on the X–variables in and out of sample are the same.

(4) The X-variables are not demeaned in these regressions.

Table 2 – Implications of Hedging for Sustainability and Welfare

		$-\overline{s}(\%)$	$Q(200)$	$Q^H(200)$	W_0/Y_0	W_0^H/Y_0	$\frac{\Delta W}{W}(\%)$
Benchmark		0.14	0.26	0.05	103.45	103.82	0.36
— full hedge		0.14	0.26	0.02	103.45	104.19	0.63
Low equity premium		0.09	0.22	0.02	121.30	122.06	0.63
— initial debt adjusted		0.09	0.42	0.33	120.88	121.37	0.40
High equity premium		0.20	0.32	0.10	86.78	87.02	0.27
— initial debt adjusted		0.20	0.14	0.01	86.97	87.14	0.19
Low hedging cost		0.09	0.26	0.04	103.45	103.98	0.38
High hedging cost		0.20	0.26	0.07	103.45	103.89	0.32
Initial Debt	$D_0/y_0 = 0$	0.14	0.03	0.001	104.06	104.32	0.25
	$D_0/y_0 = 0.15$	0.14	0.18	0.006	103.46	103.83	0.35
	$D_0/y_0 = 0.25$	0.14	0.36	0.19	103.41	103.81	0.39
	$D_0/y_0 = 0.35$	0.14	0.59	0.68	103.18	103.42	0.24
	$D_0/y_0 = 0.45$	0.14	0.83	0.95	102.81	102.96	0.15
Later initial date ($t_0 = 1986$)		0.14	0.81	0.90	97.00	97.14	0.14
Low upper bound ($\phi = .025$)		0.14	0.28	0.06	103.55	103.97	0.40
High upper bound ($\phi = .075$)		0.14	0.25	0.05	103.65	104.11	0.44
"Maastricht" Policy Rule	$(\underline{\phi}, \overline{\phi}) = (.125, .5)$	0.14	0.26	0.04	103.51	104.17	0.64
	$(\underline{\phi}, \overline{\phi}) = (.25, .5)$	0.14	0.26	0.04	103.70	104.31	0.59
Epstein–Zin	$\gamma = 4$, $\sigma = .5$	0.08	0.25	0.03	123.49	124.07	0.47
CAPM	$\gamma = 10$, $\sigma = .32$	0.11	0.25	0.04	143.74	143.11	0.44

Notes:

(1) Benchmark: $\rho_1 = -3.88$, $\rho_4 = -15.73$, t_0=1977:1, t^*=1986:2, $\phi = 0.05$, $D_0/y_0 = 0.20$, $V(\tau_0)/y_0 = 0.45$.

(2) Low risk premium: $\rho_1 = -1.92$, $\rho_4 = -12.09$. High risk premium: $\rho_1 = -5.87$, $\rho_4 = -19.91$

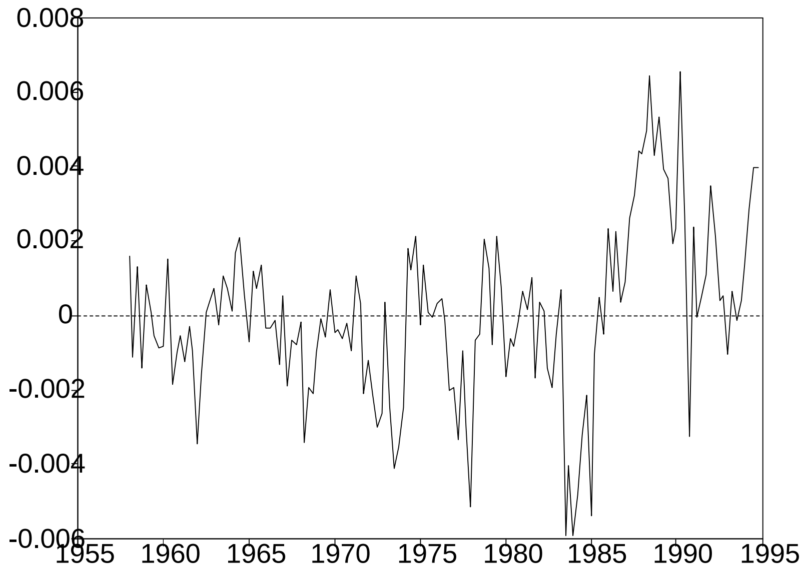

Figure1: Residuals from Surplus Regression

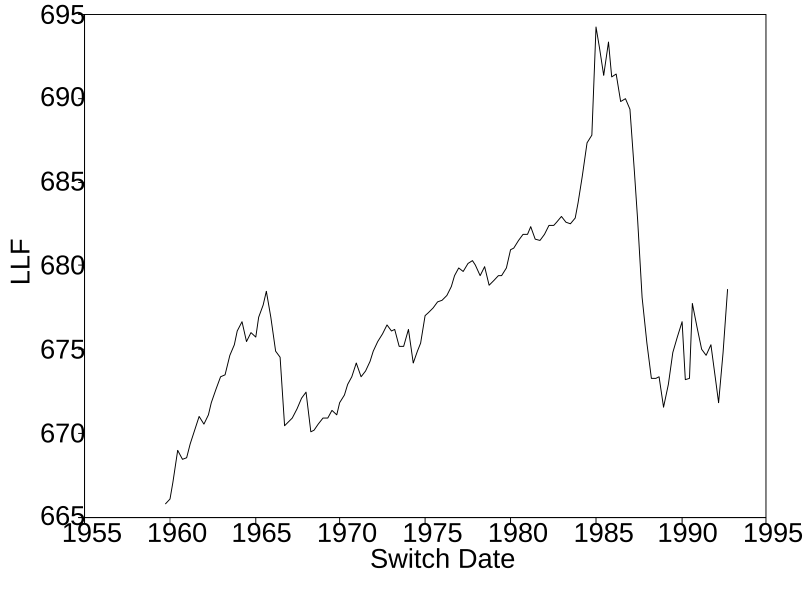

Figure 2: Maximum Likelihood Function

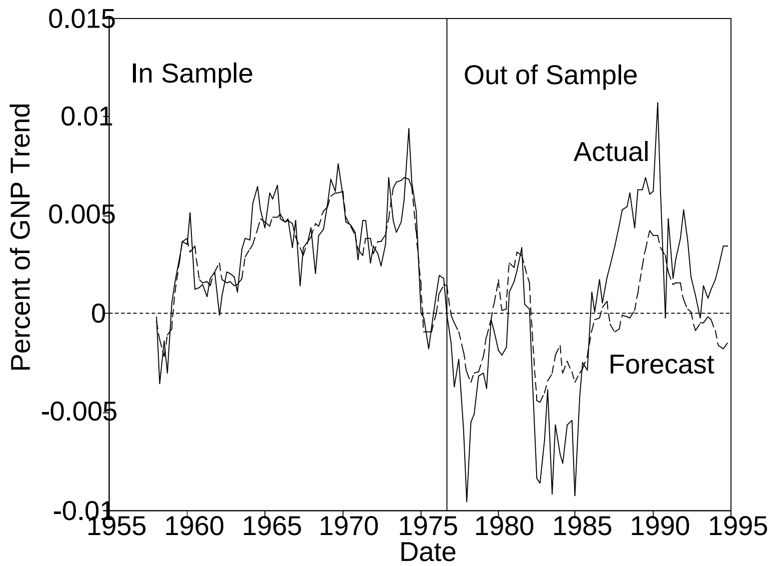

Figure 3: Actual and Forecast Primary Surplus

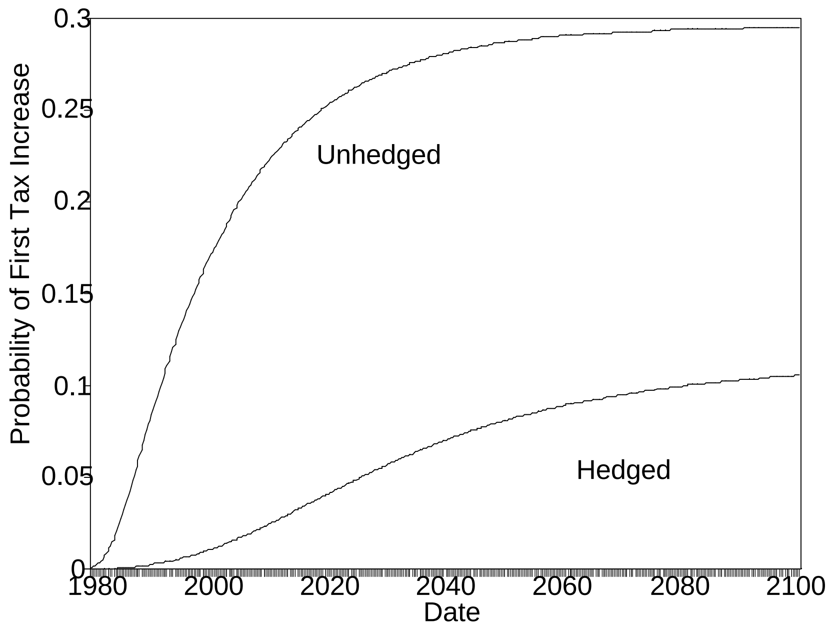

Figure 4: Sustainability of Initial Policy

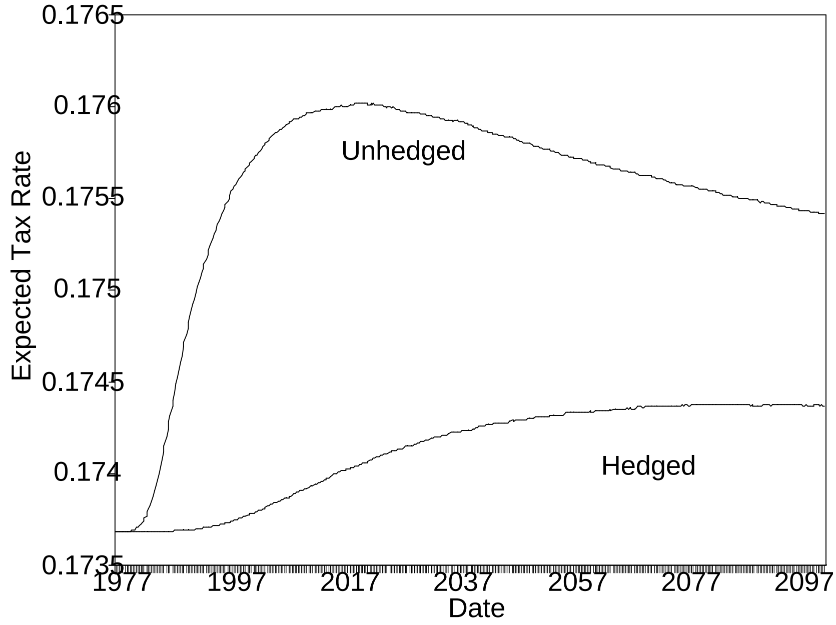

Figure 5: Expected Tax Rate