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Chain of Production as a Monetary Propagation Mechanism \*

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**Abstract:**

This paper studies a general equilibrium model with multiple stages of production and sticky prices. Working through the input-output relations among industries at different stages and the timing of firms' pricing decisions, the model generates persistent fluctuations in both the inflation rate and aggregate output following a monetary shock. The persistence is larger, the greater the number of production stages. With a sufficient number of stages, the real persistence is arbitrarily large.

Keywords:

Chain of Production; Persistence; Monetary Policy

JEL classification: E31, E32, E52

## I. INTRODUCTION

An order for a new computer often initiates a chain of orders for parts. When the order arrives at a computer vendor's desk, the vendor will start contacting suppliers of microchips, processors, hard-drives, monitors, and operating systems. The monitor maker will then contact suppliers of plastic, glass, and electronic components; and the plastic maker will respond by sending out orders to its own suppliers, and so on. The computer itself, once made, is frequently used as an intermediate input in the production of other goods.

Production of a final good typically requires multiple stages of processing. A thesis of this paper is that the multi-stage structure of production is important for explaining the relationship between money and aggregate economic activity. We show that the input-output structure helps explain persistent fluctuations in both the inflation rate and aggregate output following a monetary shock.

It is an old idea that in an industrialized economy the relationship between money, prices, and output is tied to the interdependence of firms at different stages of production. The idea has been presented at least since Means (1935). Here we quote Basu (1995):

[Means] presented evidence that different industries had very different patterns of price changes versus quantity changes in the Great Depression. Means showed that simple goods, such as agricultural products, declined heavily in price, while their quantity was almost unchanged. Complex manufactured goods, on the other hand, showed the opposite pattern, with small price changes and consequently huge declines in the quantity of sales. Crude manufactured goods fell somewhere in between.

More recent studies have confirmed Means's finding on the patterns of price changes at different stages of production (e.g., Gordon (1981), Blanchard (1987), Clark (1999), and Hanes (1999)).

The evidence presented by Means (1935) and others have led many to speculate that there are connections between the chain structure of production and aggregate fluctuations. For example, Gordon (1990) considers "the input-output table as an essential component in the description of price stickiness." Yet, few attempts have been made to theorize the idea, with the notable exception of Blanchard (1983). Blanchard (1983) shows that a simple structural model incorporating a chain of production and sticky prices can generate patterns of price changes similar to those noted by Means (1935).

Blanchard (1983) was concerned with explaining the sluggish adjustment of the price level. More recently, another set of empirical facts has attracted attention: the persistent responses of the inflation rate and aggregate output to a monetary shock (e.g., Christiano, et al. (1999)). Nelson (1998) compares the ability of several popular business cycle models with sticky prices in generating the inflation persistence. His finding suggests that most sticky price models need to be modified “to reconcile them with the actual behavior of inflation.” On the other hand, Chari, et al. (1998) challenge the ability of traditional models with staggered price contracts in the spirit of Taylor (1980) in explaining the output persistence. In meeting this challenge, various mechanisms have been proposed, most of which focus on introducing factor market frictions in the baseline model of Chari, et al. (1998) (e.g., Huang and Liu (1998) and Gust (1998)).

In this paper, we propose a new mechanism to explain the behavior of the inflation rate and aggregate output following a monetary shock. We build a model in which the production of a final good goes through multiple stages, as in Blanchard (1983), but in which individuals optimize. In the model, a firm at the first stage uses labor as an input, while a firm at a later stage uses all outputs produced at the previous stage. A representative household consumes a basket of goods produced at the final stage and supplies labor to firms at the first stage. To generate real effects of a monetary shock, we assume that pricing decisions are staggered at each stage (e.g., Taylor (1980, 1999)). We derive firms’ optimal pricing decision rules within the standard monopolistic competition framework (e.g., Blanchard and Kiyotaki (1987)). Working through the input-output relations among industries across stages and the timing of pricing decisions among firms within each stage, the model generates persistent responses of both the inflation rate and aggregate output to a monetary shock. The persistence is larger, the greater the number of production stages. If the production of a good goes through a sufficient number of stages, arbitrary real persistence obtains.

To illustrate the importance of the input-output structure in generating persistence, we first show that, in the special case of a single production stage, there is neither inflation nor real effects of money beyond the initial contract duration. In a single-stage model, prices immediately rise following the shock, since the wage rate, and hence the marginal cost for all firms, rises quickly (e.g., Chari, et al. (1998)). To generate persistence in the inflation rate or in real output, more production stages are needed.

Our baseline model with multiple stages of production does generate persistent fluctuations in both the inflation rate and real output, since firms at more advanced processing stages face smaller changes in marginal cost and thus have less incentive to change prices than do firms at less advanced stages. Following the shock, the

marginal cost for firms at the first stage immediately rises and consequently these firms raise prices fully whenever they have the chance to renew contracts. But firms at the second stage do not face the full rise in marginal cost. Their marginal cost does not rise fully because it is determined by the price index of the first-stage goods, and the price index records both the prices newly adjusted and those fixed by previous contracts. Thus firms at the second stage do not have an incentive to adjust their prices fully even if they have the chance to renew contracts. In consequence, firms at the third stage face an even smaller change in their marginal cost, and they have even less incentive to adjust prices, and so on. It turns out that, when there are more processing stages, price level adjustments become more sluggish and the responses of the inflation rate and real output to the shock become more persistent. With a sufficient number of stages, the real persistence is arbitrarily large.<sup>1</sup>

Our conclusion that the degree of price stickiness is a function of the number of production stages is similar to that of Blanchard (1983), but for different reasons. In his model, pricing decisions are staggered across different stages and firms within each stage are homogeneous. Basu (1995) points out that, “if the pricing decision in Blanchard’s model were made state-dependent then, since the ‘first good’ is made without intermediate goods, there would be no increase in price rigidity regardless of the number of stages of production.” But Basu’s (1995) criticism does not apply to our model. In our model, pricing decisions are staggered among firms *within* each stage. Under a state-dependent pricing rule, firms at each stage in general do not have an incentive to synchronize as long as they face heterogeneous costs of changing prices (e.g., Dotsey, et al. (1997, 1999)). Thus, changes in marginal cost will diminish along the production chain and price rigidity will increase with the number of stages.<sup>2</sup>

There is also some similarity between our model and that of Basu (1995), both suggesting that a small rigidity in prices of intermediate goods generates large real effects of a monetary shock. Yet, the models differ in two aspects. First, Basu (1995) assumes pricing decisions are state-dependent, while in our model, they are time-dependent. As we have just noted, our results are robust under state-dependent pricing rules. Second, and more importantly, the input-output structures differ. Basu (1995) assumes a single production stage with a roundabout input-output structure, while we have multiple stages of processing with an in-line chain-of-production structure. Both types of input-output structure are empirically relevant. While Basu (1995) has shown that a roundabout input-output structure is an important source of real rigidity, we demonstrate here that the chain structure of production plays an important role in propagating monetary shocks.

The assumption that pricing decisions are staggered is supported by empirical evidence (e.g., Taylor (1999)). Yet, answering the question of why there is stagger-

ing rather than complete synchronization is beyond the scope of this paper. In the literature, some progress has been made on this issue. Dotsey, et al. (1997) show that introducing heterogeneity of menu costs across firms can result in endogenous staggering. Ball and Romer (1989) demonstrate that staggering is an equilibrium outcome if there are firm-specific shocks that arrive at different time for different firms. Ball and Cecchetti (1988) show that, with imperfect information, firms cannot distinguish between aggregate demand shocks and firm-specific shocks, and thus do not have an incentive to synchronize. Gordon (1990) argues that, in a world with imperfect information, the complexity of the input-output table makes it unlikely for firms to synchronize, since “the typical firm has no idea of the identity of its full set of suppliers when all the indirect links within the input-output table are considered. . . . [T]he sensible firm just waits by the mailbox for news of cost increases and then . . . passes them on as price increases.” Clearly, incorporating these elements and thus making staggering endogenous will make the model more intuitively appealing. But it will not change the mechanism through which the production chain propagates monetary shocks.

The assumption that labor market is perfectly competitive is for the purpose of isolating the role of the input-output structure in transmitting monetary shocks. Under this assumption, labor costs change quickly following a shock, creating an incentive for a quick price adjustment. Thus, any price level rigidity is generated solely through the input-output structure. Incorporating labor market rigidity will dampen fluctuations of labor costs and therefore, along with the input-output interactions, will generate more sluggish changes in the price level. In this sense, adding labor market rigidity strengthens our results.

In what follows, we describe the model in Section II, present the results in Section III, and conclude the paper in Section IV. All proofs are contained in the Appendix.

## II. THE MODEL

In the model, production of consumption goods requires  $N$  stages of processing, from crude material to intermediate goods, then to more advanced goods, and so on. At each stage, there is a continuum of firms indexed in the interval  $[0, 1]$ , producing differentiated goods. Production at stage  $n \in \{2, \dots, N\}$  requires all goods produced at stage  $n - 1$ , while production at the first stage (i.e., the raw material sector) uses homogeneous labor services provided by a representative household (see Figure I for an illustration of the economy’s structure).

In each period  $t$ , there realizes a shock  $s_t$ . The history of events up to date  $t$  is  $s^t \equiv (s_0, \dots, s_t)$ , with probability  $\pi(s^t)$ . The initial realization  $s_0$  is given.

The representative household is infinitely lived and has an expected lifetime utility given by

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) \left[ \ln C(s^t) + \Phi \ln \left( \frac{M(s^t)}{\bar{P}_N(s^t)} \right) - \Psi L(s^t) \right],$$

where  $\beta \in (0, 1)$  is a subjective discount factor,  $C(s^t)$  is consumption,  $M(s^t)$  is nominal money balances,  $L(s^t)$  is labor hours, and  $\bar{P}_N(s^t)$  is a price index of goods produced at the final stage (i.e., a price level). The consumption good is a Dixit-Stiglitz (1977) composite of final-stage goods. Specifically, we have

$$(1) \quad C(s^t) = \left[ \int_0^1 Y_N(i, s^t)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}} \equiv Y(s^t),$$

where  $Y_N(i, s^t)$  is a type  $i$  good produced at stage  $N$  and  $\theta$  is an elasticity of substitution among all such goods. The household is endowed with one unit of time in each period. It faces a sequence of budget constraints

$$(2) \quad \int_0^1 P_N(i, s^t) Y_N(i, s^t) di + \sum_{s^{t+1}} D(s^{t+1}|s^t) B(s^{t+1}) + M(s^t) \\ \leq W(s^t) L(s^t) + \Pi(s^t) + B(s^t) + M(s^{t-1}) + T(s^t),$$

where  $P_N(i, s^t)$  is the price of a type  $i$  good produced at the final stage,  $B(s^{t+1})$  is a one-period nominal bond that costs  $D(s^{t+1}|s^t)$  dollars at  $s^t$  and pays off one dollar in the next period if  $s^{t+1}$  is realized,  $W(s^t)$  is a nominal wage,  $\Pi(s^t)$  is the household's claim to all firms' profits, and  $T(s^t)$  is a nominal lump-sum transfer from the government. The household maximizes utility subject to (1), (2), and a borrowing constraint  $B(s^t) \geq -\bar{B}$  for some large positive  $\bar{B}$ , taking the wage and prices as given. The initial conditions  $M(s^{-1})$  and  $B(s^0)$  are also taken as given.

The price index  $\bar{P}_N(s^t)$  is given by  $\bar{P}_N(s^t) = \left[ \int_0^1 P_N(i, s^t)^{1-\theta} di \right]^{\frac{1}{1-\theta}}$ . It follows that the expenditure on the basket of consumption goods equals the total expenditure on all types of goods produced at the final stage, that is,  $\bar{P}_N(s^t) Y(s^t) = \int_0^1 P_N(i, s^t) Y_N(i, s^t) di$ . The demand function for a type  $i$  good produced at stage  $N$  is

$$(3) \quad Y_N^d(i, s^t) = \left[ \frac{P_N(i, s^t)}{\bar{P}_N(s^t)} \right]^{-\theta} Y(s^t).$$

Thus, the more expensive is good  $i$  relative to other stage- $N$  goods, the lower is the relative demand for  $i$ .

Production of each good at stage  $n \in \{2, \dots, N\}$  requires all goods produced at the previous stage. Specifically, the production function is

$$(4) \quad Y_n(i, s^t) = \left[ \int_0^1 Y_{n-1}(i, j, s^t)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}},$$

where  $Y_n(i, s^t)$  is the output of firm  $i$  at stage  $n$  and  $Y_{n-1}(i, j, s^t)$  is the input supplied by firm  $j$  at stage  $n-1$ . Production of each good at the first stage uses labor, with a constant returns to scale production function  $Y_1(i, s^t) = L(i, s^t)$ , where  $Y_1(i, s^t)$  is the output and  $L(i, s^t)$  is the labor input.

To generate real effects of monetary shocks, we assume that pricing decisions are staggered (e.g., Taylor (1980, 1999)), and we derive optimal pricing decision rules within a monopolistic competition framework (e.g., Blanchard and Kiyotaki (1987)). To focus on the role of the chain-of-production in propagating monetary shocks, we look at simple two-period staggered price contracts. Under such contracts, in each period, half of the firms at each stage can set new prices for their outputs. Once a price is set, it remains effective for two periods, which is referred to as a ‘‘contract duration’’. We sort the indices of firms at each stage so that those indexed  $i \in [0, 1/2]$  set new prices in periods 0, 2, 4,  $\dots$ , and those indexed  $i \in (1/2, 1]$  set new prices in periods 1, 3, 5,  $\dots$ , and so on. Formally, upon the realization of  $s^t$ , if firm  $i$  at stage  $n \in \{1, \dots, N\}$  can set a new price, it chooses  $P_n(i, s^t)$  to solve

$$(5) \quad \text{Max} \quad \sum_{\tau=t}^{t+1} \sum_{s^\tau} D(s^\tau | s^t) [P_n(i, s^t) - V_n(i, s^\tau)] Y_n^d(i, s^\tau),$$

taking its unit cost function  $V_n(i, s^\tau)$  and a demand schedule  $Y_n^d(i, s^\tau)$  as given.

The unit cost for firms at the first stage is simply the nominal wage rate, that is,  $V_1(s^\tau) \equiv V_1(i, s^\tau) = W(s^\tau)$ , since labor is the only input at that stage. The unit cost for firms at stage  $n \in \{2, \dots, N\}$  is derived from minimizing the cost  $\int_0^1 P_{n-1}(j) Y_{n-1}(i, j) dj$  subject to (4). The resulting unit cost is  $V_n(s^\tau) \equiv V_n(i, s^\tau) = \bar{P}_{n-1}(s^\tau)$ , where  $\bar{P}_{n-1}(s^\tau) \equiv \left[ \int_0^1 P_{n-1}(j, s^\tau)^{1-\theta} dj \right]^{\frac{1}{1-\theta}}$  is a price index of all goods produced at stage  $n-1$ . Given constant-returns-to-scale technologies, the unit cost is also the marginal cost and it is firm-independent.

In the case with  $n \in \{2, \dots, N\}$ , the firm’s demand for good  $j$  produced at stage  $n-1$  is also derived from the cost-minimization problem and is given by

$$Y_{n-1}^d(i, j, s^\tau) = \left[ \frac{P_{n-1}(j, s^\tau)}{\bar{P}_{n-1}(s^\tau)} \right]^{-\theta} Y_n(i, s^\tau).$$

The total demand for good  $j$  is the sum of the demand by all firms at stage  $n$ , that is,

$$(6) \quad Y_{n-1}^d(j, s^\tau) = \left[ \frac{P_{n-1}(j, s^\tau)}{\bar{P}_{n-1}(s^\tau)} \right]^{-\theta} Y_n(s^\tau),$$

where  $Y_n(s^\tau) \equiv \int_0^1 Y_n(i, s^\tau) di$  is a linear aggregate of stage- $n$  outputs. Equation (6) says that the demand for  $j$  is higher if its price relative to the price index of all stage- $(n-1)$  goods is lower.

Solving the profit-maximization problem (5) yields the optimal pricing decision rule

$$(7) \quad P_n(i, s^t) = \frac{\theta}{\theta - 1} \frac{\sum_{\tau=t}^{t+1} \sum_{s^\tau} D(s^\tau | s^t) Y_n^d(i, s^\tau) V_n(s^\tau)}{\sum_{\tau=t}^{t+1} \sum_{s^\tau} D(s^\tau | s^t) Y_n^d(i, s^\tau)},$$

where  $n \in \{1, \dots, N\}$ . Thus the optimal price is a constant mark-up over a weighted average of the firm's marginal costs within the contract duration. The weights are normalized total demand for its output. In light of (3) and (6), the weights depend on industry- and economy-wide variables only. If the expected marginal costs rise, the firm will respond by raising its price.

A monetary authority injects newly created money into the economy via a lump-sum transfer to the household, so that

$$(8) \quad T(s^t) = M^s(s^t) - M^s(s^{t-1}).$$

The money supply  $M^s(s^t)$  grows at a rate  $\mu(s^t)$ , that is,  $M^s(s^t) = \mu(s^t)M^s(s^{t-1})$ . We assume that  $\ln \mu(s^t)$  follows a stationary stochastic process.

An *equilibrium* for this economy consists of allocations  $\{Y_N(i, s^t)\}_{i \in [0,1]}$ ,  $L(s^t)$ ,  $M(s^t)$ , and  $B(s^{t+1})$  for the household, allocations  $\{L(i, s^t)\}_{i \in [0,1]}$  and prices  $\{P_1(i, s^t)\}_{i \in [0,1]}$  for firms at the first stage, allocations  $\{Y_{n-1}(i, j, s^t)\}_{i, j \in [0,1]}$  and prices  $\{P_n(i, s^t)\}_{i \in [0,1]}$  for firms at stage  $n$ , for every  $n \in \{2, \dots, N\}$ , and wage rate  $W(s^t)$ , bond prices  $D(s^{t+1} | s^t)$ , and price indices  $\{\bar{P}_n(s^t)\}_{n \in \{1, \dots, N\}}$  that satisfy the following conditions: (i) taking wage and prices as given, the household's allocations solve the utility maximization problem; (ii) taking wage and all prices but its own as given, each firm's allocation and price solve its profit maximization problem; (iii) markets for labor, money, and bonds clear; (iv) money supply and transfers are as specified.

It is important to recognize that the composite of final goods  $Y(s^t)$  in (1) can be interpreted as an aggregate output, corresponding to real GDP in the data.<sup>3</sup> To justify this interpretation, first observe that, in an equilibrium, the budget constraint (2) is binding since the utility function is strictly monotone. Then, by imposing the money market clearing condition and the transfer process (8), we can

cancel out the terms involving money balances and transfers in the budget equation. From the bond market clearing condition (i.e.,  $B(s^t) = 0$ ), the terms involving nominal bonds drop out. Thus, with the equilibrium relation  $\bar{P}_N(s^t)Y(s^t) = \int_0^1 P_N(i, s^t)Y_N(i, s^t)di$ , the budget equation reduces to

$$\bar{P}_N(s^t)Y(s^t) = W(s^t)L(s^t) + \Pi(s^t).$$

The left-hand side of the equation is the aggregate expenditure while the right-hand side is the total income, including wage income and equity income. The equity income is the total profits of firms at all production stages. Thus the right-hand side is also the aggregate value-added. It is clear from this equation that  $Y(s^t)$  corresponds to real aggregate output, or real GDP.

We focus on a symmetric equilibrium in which firms in the same cohort make identical decisions. In a symmetric equilibrium, firms are identified by the stage at which they produce and the time at which they can change prices. Thus we drop the indices  $i$  and  $j$  for individual firms, and let  $P_n(t)$  denote prices set at time  $t$  for goods produced at stage  $n \in \{1, \dots, N\}$ .

### III. THE RESULTS

In this section, we show that the chain structure of production helps explain the persistent responses of the inflation rate and aggregate output to a monetary shock. The persistence increases with the number of production stages. When there is a sufficient number of stages, arbitrary real persistence obtains.

To elaborate the results, we derive analytical solutions to a log-linearized system of equilibrium conditions. We begin by reducing the equilibrium conditions to  $2N + 2$  equations, including  $N$  pricing decision equations, a labor supply decision equation, a money demand equation, and  $N$  equations defining price indices. We then log-linearize these equations around a deterministic steady state. In what follows, we use lowercase letters to denote the log-deviations of the corresponding level variables from their steady state values.<sup>4</sup>

The linearized pricing decision rule for firms at stage  $n \in \{1, \dots, N\}$  is given by

$$(9) \quad p_n(t) = \frac{1}{1 + \beta} \bar{p}_{n-1}(t) + \frac{\beta}{1 + \beta} E_t[\bar{p}_{n-1}(t + 1)],$$

where  $\bar{p}_0(t)$  denotes the nominal wage  $w(t)$  and  $E_t$  is a conditional expectation operator. According to (9), a firm's optimal price is a weighted average of its expected marginal costs within the contract duration. The marginal cost for a firm at stage  $n \in \{2, \dots, N\}$  is the price index of goods produced at stage  $n - 1$  since

the firm uses all these goods as inputs. The marginal cost for a firm at the first stage is the nominal wage since labor is the only input of that stage. If the marginal costs are expected to rise, a firm will respond by setting a higher price if it can renew its contract.

The labor supply decision of the household is described by

$$(10) \quad w(t) = \bar{p}_N(t) + y(t).$$

Thus real wage is proportional to aggregate output. The money demand equation is

$$(11) \quad \bar{p}_N(t) + y(t) = (1 - \beta)m(t) + \beta E_t[\bar{p}_N(t + 1) + y(t + 1)].$$

Therefore, nominal GDP is a weighted average of money and expected future nominal GDP. The presence of the expectation terms in (11) reveals that the money demand is interest-rate sensitive.

Finally, the price index at stage  $n \in \{1, \dots, N\}$  is related to pricing decisions by

$$(12) \quad \bar{p}_n(t) = \frac{1}{2}p_n(t - 1) + \frac{1}{2}p_n(t).$$

Under the staggered contracts, the price index at each stage records both the price set in the current period and that set in the previous period. The lagged price enters (12) because each contract lasts for two periods.

The equilibrium conditions are fully described by (9)-(12). To focus on the role of the input-output structure in generating persistence in the inflation rate and aggregate output, we assume that there is no serial correlation in the money growth process. In particular, we assume that the money supply follows a random walk process, i.e.,  $m(t) = m(t - 1) + \epsilon(t)$ , where  $\epsilon(t)$  is a white noise disturbance corresponding to the money growth rate. Suppose that there is a one percent shock to the money growth rate in period 0, that is,  $\epsilon(0) = 1$  and  $\epsilon(t) = 0$  for all  $t \geq 1$ . We compute the impulse response functions to determine how the shock is divided between movements in the price level and in aggregate output. Thus, we focus on a perfect foresight equilibrium and drop the expectation operator  $E_t$ . The following proposition partially characterizes the equilibrium.

**PROPOSITION 3.1:** *There is a unique perfect foresight equilibrium in which*

$$(13) \quad w(t) = 1, \quad t \geq 0,$$

$$(14) \quad p_n(t) = 1, \quad t \geq n - 1, \quad n \in \{1, \dots, N\},$$

$$(15) \quad \bar{p}_n(t) = 1, \quad t \geq n, \quad n \in \{1, \dots, N\},$$

$$(16) \quad y(t) = 0, \quad t \geq N.$$

Following the shock, nominal wage immediately rises, so does the marginal cost for firms at the first stage. These firms thus fully raise their prices whenever they can renew contracts. At the end of the initial contract duration when they all have had a chance to change prices, the first-stage price index is entirely composed of fully raised prices and thus rises fully as well.

In the case of a single production stage (i.e.,  $N = 1$ ), there is neither inflation nor real effects of money beyond the initial contract duration since, in this case, the price level corresponds to the first-stage price index which rises fully as soon as the initial contract duration is over. Clearly, to obtain a persistent response of the inflation rate or of real output, a sluggish adjustment of the price level is necessary. We now demonstrate that, with multiple production stages, the model does generate such a sluggish adjustment.

**PROPOSITION 3.2:** *Suppose  $N \geq 2$ . In the perfect foresight equilibrium, the strict inequalities*

$$(17) \quad p_{n+1}(t) < p_n(t), \quad 0 \leq t \leq n-1,$$

$$(18) \quad \bar{p}_{n+1}(t) < \bar{p}_n(t), \quad 0 \leq t \leq n$$

*hold for every  $n \in \{1, \dots, N-1\}$ .*

Hence, when the number of stages increases, changes in the price level are smaller on a period-by-period basis and the price level does not rise fully for more periods. In other words, the greater the number of stages, the more sluggish is the adjustment of the price level.

The key to understanding this result is to see how the effects of the shock on marginal costs are gradually dampened through the chain. The dampening process is illustrated in Figure II for the case with  $N = 2$  (the arrows in the figure correspond to the equilibrium relations between price decisions and price indices described by (9) and (12)). Following the shock, firms at the first stage face a full rise in marginal cost and consequently raise their prices fully whenever they can renew contracts. Firms at the second stage, however, do not face the full rise in marginal cost until the second period arrives. The marginal cost of these firms is equal to the first-stage price index. In the impact period, this price index is an average of the prices newly adjusted and those fixed by contracts and therefore, does not rise fully. Facing a partial increase in marginal cost, firms at the second stage choose not to raise their prices fully even if they can set new prices. At the end of the initial contract duration, the first-stage price index rises fully, so does

the marginal cost for firms at the second stage. Thus, those firms that can renew contracts do choose to adjust prices accordingly. Yet, the second-stage price index does not rise fully because it is an average of the prices newly adjusted and those partially adjusted in the impact period. In consequence, changes in prices at the second stage are smaller and less rapid than do changes in prices at the first stage, and the price level does not rise fully even when the initial contract duration is over.

When  $N$  becomes larger, the impact of the shock on marginal costs diminishes from earlier to later stages, and the adjustments in the price level become more sluggish. In particular, the price level does not rise fully until period  $N$  arrives, as illustrated by Table I.

#### A. Inflation Persistence

In light of Proposition 3.2, a greater number of production stages corresponds to more sluggish changes in the price level. Thus, inflation will last for more periods. In addition, the equilibrium relations (10) and (13) suggest that, if adjustments of the price level are more sluggish, then the response of real output will be larger on a period-by-period basis and be longer-lasting. This finding opens the way for the chain-of-production mechanism to generate persistent effects of the shock on inflation and real output.

Yet, to have more persistent responses of the inflation rate and real output also requires higher auto-correlations in these variables so as to allow their impulse responses to die out more gradually following the shock. In other words, it requires larger impulse responses in later periods *relative to* those in earlier periods. Based on this idea, we measure the magnitude of persistence by the ratio of the impulse response in period  $t$  to that in period  $t - 1$ .<sup>5</sup>

We now establish the monotonicity of inflation persistence in the number of stages. With  $N$  stages, the inflation rate in period  $t$  is equal to  $\bar{p}_N(t) - \bar{p}_N(t - 1)$ . The inflation persistence is monotone if the ratio of the inflation rate in period  $t$  to that in period  $t - 1$  is increasing in  $N$ .

**PROPOSITION 3.3 (Monotonicity of inflation persistence):** *In the perfect foresight equilibrium, the strict inequality*

$$(19) \quad \frac{\bar{p}_{N+1}(t) - \bar{p}_{N+1}(t - 1)}{\bar{p}_{N+1}(t - 1) - \bar{p}_{N+1}(t - 2)} > \frac{\bar{p}_N(t) - \bar{p}_N(t - 1)}{\bar{p}_N(t - 1) - \bar{p}_N(t - 2)}, \quad 1 \leq t \leq N + 1,$$

*holds for all  $N \geq 1$ .*

Thus the greater is the number of stages, the more gradually does the response of the inflation rate die out.

### B. Output Persistence

Given the equilibrium relations in (10) and (13), real aggregate output in period  $t$  is equal to  $1 - \bar{p}_N(t)$  when there are  $N$  stages. Output persistence is monotone if the ratio of output in period  $t$  to that in period  $t - 1$  increases with  $N$ .

PROPOSITION 3.4 (Monotonicity of output persistence): *In the perfect foresight equilibrium, the strict inequality*

$$(20) \quad \frac{1 - \bar{p}_{N+1}(t)}{1 - \bar{p}_{N+1}(t-1)} > \frac{1 - \bar{p}_N(t)}{1 - \bar{p}_N(t-1)}, \quad 1 \leq t \leq N,$$

*holds for all  $N \geq 1$ .*

Therefore, the greater the number of stages, the more persistent is the response of output to the shock. To illustrate this result, we examine the model's implications on output persistence based on two persistence measures which are special cases of ours. One is the ratio of the output response at the end of the initial contract duration to that in the impact period (i.e., the “contract multiplier”). The other is the number of periods it takes for output to return to half of the level of its initial response (i.e., the “half-life”). As illustrated by Table II, both the contract multiplier and the half-life increase with  $N$ . As the number of stages grows from one to five and then to ten, for example, the contract multiplier increases from 0 to 0.46 and then to 0.62.

The remaining question is: how long a way can the input-output structure go in generating real persistence? Our next result shows that, when  $N$  is sufficiently large, the ratio of the output response in period  $t$  to that in period  $t - 1$  is sufficiently close to 1.

PROPOSITION 3.5: *In the perfect foresight equilibrium, the equality*

$$(21) \quad \lim_{N \rightarrow \infty} \frac{1 - \bar{p}_N(t)}{1 - \bar{p}_N(t-1)} = 1$$

*holds for all  $t \geq 1$ .*

According to (21), arbitrary real persistence obtains when there is a sufficient number of stages. In the proof of this proposition, we show that when  $N$  approaches infinity, the price level does not change and real output carries the full burden of adjustment. Thus the chain-of-production mechanism goes a long way in generating real persistence.

#### IV. CONCLUSION

We have shown that a model with multiple stages of production and sticky prices helps explain the behavior of inflation and aggregate output following a monetary shock. The effects of the shock on the inflation rate and real output are more persistent, the greater the number of production stages. With a sufficient number of stages, arbitrary real persistence obtains.

To help exposition, we have assumed a dense input-output structure, with labor being used at the first stage only. These assumptions are not essential for our results. Our conclusion does not hinge upon the assumption that production of a good at a given stage uses *all* outputs produced at the previous stage. To dampen the fluctuations of marginal costs across stages, what matters is that input-supplying firms do not change their prices simultaneously. It does not matter whether the input-supplying firms constitute all or just a fraction of the firms of the previous stage. Neither do our results depend on the assumption that labor is used at the first stage only. With labor input at every stage, the mechanism through which marginal cost fluctuations are dampened along the production chain works in the same way as in the baseline model.

To assess the quantitative importance of the input-output structure in explaining the relationship between money, prices, and output, however, we do need to have labor input at every stage and to calibrate the share of labor and of intermediate goods at each stage. A sensible quantitative model built for this purpose should also take into account labor market rigidity, for there is overwhelming evidence on such rigidity. In a model like this, labor and purchased materials are both a component of cost. With nominal rigidity in both the labor market and the goods market and through the interactions of prices and costs along the production chain, the model is likely to account for a significant fraction of the observed fluctuations in the inflation rate and real output following a monetary shock.<sup>6</sup>

The quantitative importance of the input-output structure also depends on the number of production stages (the  $N$  in our model). Calibrating the value of  $N$ , however, requires a detailed examination of the input-output table. In light of our conclusion that the input-output structure is potentially a powerful mechanism in propagating monetary shocks, an empirical investigation of the input-output table should be elevated to the top of the research agenda. Casual observations do suggest that  $N$  is likely to be large. On this, Gordon (1990) pictures the world as “a gigantic  $n \times n$  matrix, where  $n$  is measured in the thousands, if not the millions. . . . The gigantic matrix represents the real world, full of heterogeneous firms enmeshed in a web of intricate supplier-demander relationships.” In this web, the intricately made computer is perhaps just a tiny node.

APPENDIX: PROOFS

PROOF OF PROPOSITION 3.1: Using (10), (11) and  $m(t) = 1$  for  $t \geq 0$ , we obtain

$$(22) \quad w(t) = 1 + [w(0) - 1]/\beta^t$$

for each  $t \geq 0$ . Substituting (22) into (9) for the case with  $n = 1$  yields

$$(23) \quad p_1(t) = 1 + 2[w(0) - 1]/[\beta^t(1 + \beta)]$$

for each  $t \geq 0$ . Substituting (12) into (9) leads to

$$(24) \quad p_n(t) = \frac{1}{2(1 + \beta)}p_{n-1}(t - 1) + \frac{1}{2}p_{n-1}(t) + \frac{\beta}{2(1 + \beta)}p_{n-1}(t + 1)$$

for each  $t \geq 0$  and each  $n \in \{2, \dots, N\}$ . Using (23) and (24), we can prove by induction on  $n$  that

$$(25) \quad p_n(t) = 1 + 2[w(0) - 1]/[\beta^t(1 + \beta)]$$

for each  $t \geq n - 1$  and each  $n \in \{2, \dots, N\}$ . It then follows from (12), (23) and (25) that

$$(26) \quad \bar{p}_n(t) = w(t)$$

for each  $t \geq n$  and each  $n \in \{1, \dots, N\}$ .

We claim that the only value of  $w(0)$  that is consistent with an equilibrium is  $w(0) = 1$ . If otherwise,  $w(0) > 1$  or  $w(0) < 1$ , then by (22), as  $t$  goes to infinity,  $w(t)$  diverges to plus or minus infinity at a rate of  $1/\beta$ , so does the price level  $\bar{p}_N(t)$  as implied by (26). These possibilities, however, can be ruled out as in Obstfeld and Rogoff (1983, 1986). The hyper-inflationary path with  $w(t) \rightarrow \infty$  cannot be an equilibrium, because with the log-utility in real balances the household would suffer an infinite utility loss as real balances approach zero along such a path. The hyper-deflationary path with  $w(t) \rightarrow -\infty$  cannot be an equilibrium either, because it would violate the appropriate transversality condition with respect to real balances. Therefore,  $w(0) = 1$ , and there is a unique equilibrium in which  $w(t) = 1$  for all  $t \geq 0$  according to (22). That is, equation (13) holds. Equations (14) and (15) then follow from (23), (25) and (26). Finally, equation (16) follows from (10), (13) and (15). This completes the proof.

PROOF OF PROPOSITION 3.2: We prove (17) by induction on  $n$ . We first verify (17) for  $n = 1$ . Equation (14) implies that  $p_1(0) = 1$  and thus  $\bar{p}_1(0) = 1/2$

according to (12). This together with  $\bar{p}_1(1) = 1$  by (15) results in  $p_2(0) = 1 - 1/[2(1 + \beta)] < 1$  according to (9). Therefore, (17) holds for  $n = 1$ . This would be the end of the proof of (17) if  $N = 2$ . Without loss of generality, we assume  $N > 2$ . Suppose that (17) holds for  $n$  with  $1 \leq n \leq N - 2$ . We need to show that (17) holds for  $n + 1$ , that is,

$$(27) \quad p_{n+2}(t) < p_{n+1}(t), \quad 0 \leq t \leq n.$$

Fix an arbitrary  $t$  with  $0 \leq t \leq n$ . It follows that  $-1 \leq t - 1 \leq n - 1$  and  $1 \leq t + 1 \leq n + 1$ . By the induction hypothesis and (14), we have

$$p_{n+1}(t - 1) \leq p_n(t - 1), \quad p_{n+1}(t) \leq p_n(t), \quad p_{n+1}(t + 1) \leq p_n(t + 1),$$

with at least one strict inequality. Noticing that relation (24) holds for each  $t \geq 0$  and each  $n \in \{2, \dots, N\}$ , we have

$$\begin{aligned} p_{n+2}(t) - p_{n+1}(t) &= \frac{1}{2(1 + \beta)}[p_{n+1}(t - 1) - p_n(t - 1)] + \\ &\quad \frac{1}{2}[p_{n+1}(t) - p_n(t)] + \frac{\beta}{2(1 + \beta)}[p_{n+1}(t + 1) - p_n(t + 1)] < 0, \end{aligned}$$

which establishes (27). This completes the proof of (17).

To prove (18), fix an arbitrary  $n \in \{1, \dots, N - 1\}$  and an arbitrary  $t$  with  $0 \leq t \leq n$ . It follows that  $-1 \leq t - 1 \leq n - 1$ . Then (14) and (17) imply that

$$p_{n+1}(t - 1) \leq p_n(t - 1), \quad p_{n+1}(t) \leq p_n(t),$$

with at least one strict inequality, which together with (12) leads to

$$\bar{p}_{n+1}(t) - \bar{p}_n(t) = \frac{1}{2}[p_{n+1}(t - 1) - p_n(t - 1)] + \frac{1}{2}[p_{n+1}(t) - p_n(t)] < 0.$$

This establishes (18), and thus completes the proof.

**PROOF OF PROPOSITION 3.3:** We prove the proposition by induction on  $N$ . To simplify notations, we define  $\pi_N(t) \equiv \bar{p}_N(t) - \bar{p}_N(t - 1)$ . Thus  $\pi_N(t)$  is the inflation rate in period  $t$  when there are  $N$  stages. The claimed inequality in (19) can be rewritten as

$$(28) \quad \frac{\pi_{N+1}(t)}{\pi_{N+1}(t - 1)} > \frac{\pi_N(t)}{\pi_N(t - 1)}, \quad 1 \leq t \leq N + 1.$$

It is straightforward to verify that (28) holds for  $N = 1$  and  $N = 2$ . Now suppose it holds when there are  $N > 2$  stages. We need to show that it also holds when there are  $N + 1$  stages. That is, we need to establish

$$(29) \quad \frac{\pi_{N+2}(t)}{\pi_{N+2}(t-1)} > \frac{\pi_{N+1}(t)}{\pi_{N+1}(t-1)}, \quad 1 \leq t \leq N+2.$$

We proceed by first noting that, when adding an additional stage to a chain with  $N$  stages, (9)-(12) remain to be equilibrium conditions for the modified economy with  $N + 1$  stages, with  $N + 1$  in place of  $N$  everywhere.

Manipulating (9) and (12) for index  $N + 1$  leads to

$$(30) \quad \bar{p}_{N+1}(t) = \begin{cases} \frac{1}{2(1+\beta)}\bar{p}_N(t-1) + \frac{1}{2}\bar{p}_N(t) + \frac{\beta}{2(1+\beta)}\bar{p}_N(t+1), & \text{if } t \geq 1, \\ \frac{1}{2(1+\beta)}\bar{p}_N(0) + \frac{\beta}{2(1+\beta)}\bar{p}_N(1), & \text{if } t = 0, \end{cases}$$

which, along with the notation  $\pi_N(t) \equiv \bar{p}_N(t) - \bar{p}_N(t-1)$ , implies that

$$(31) \quad \pi_{N+1}(t) = \begin{cases} \frac{1}{2(1+\beta)}\pi_N(t-1) + \frac{1}{2}\pi_N(t) + \frac{\beta}{2(1+\beta)}\pi_N(t+1), & \text{if } t \geq 2, \\ \frac{1}{2}\pi_N(0) + \frac{1}{2}\pi_N(1) + \frac{\beta}{2(1+\beta)}\pi_N(2), & \text{if } t = 1, \\ \frac{1}{2}\pi_N(0) + \frac{\beta}{2(1+\beta)}\pi_N(1), & \text{if } t = 0. \end{cases}$$

Similarly, the  $\pi_{N+2}(t)$  term in (29) is given by

$$(32) \quad \pi_{N+2}(t) = \begin{cases} \frac{1}{2(1+\beta)}\pi_{N+1}(t-1) + \frac{1}{2}\pi_{N+1}(t) + \frac{\beta}{2(1+\beta)}\pi_{N+1}(t+1), & \text{if } t \geq 2, \\ \frac{1}{2}\pi_{N+1}(0) + \frac{1}{2}\pi_{N+1}(1) + \frac{\beta}{2(1+\beta)}\pi_{N+1}(2), & \text{if } t = 1, \\ \frac{1}{2}\pi_{N+1}(0) + \frac{\beta}{2(1+\beta)}\pi_{N+1}(1), & \text{if } t = 0. \end{cases}$$

By the induction hypothesis, we have

$$(33) \quad \frac{\pi_{N+1}(t+1)}{\pi_{N+1}(t)} > \frac{\pi_N(t+1)}{\pi_N(t)}, \quad \frac{\pi_{N+1}(t)}{\pi_{N+1}(t-1)} > \frac{\pi_N(t)}{\pi_N(t-1)}, \\ \frac{\pi_{N+1}(t-1)}{\pi_{N+1}(t-2)} > \frac{\pi_N(t-1)}{\pi_N(t-2)}, \quad \text{if } 2 \leq t \leq N;$$

$$(34) \quad \pi_{N+1}(t+1) = \pi_N(t+1) = \pi_N(t) = 0, \quad \frac{\pi_{N+1}(t)}{\pi_{N+1}(t-1)} > \frac{\pi_N(t)}{\pi_N(t-1)}, \\ \frac{\pi_{N+1}(t-1)}{\pi_{N+1}(t-2)} > \frac{\pi_N(t-1)}{\pi_N(t-2)}, \quad \text{if } t = N+1;$$

$$(35) \quad \begin{aligned} \pi_{N+1}(t+1) &= \pi_{N+1}(t) = \pi_N(t+1) = \pi_N(t) = \pi_N(t-1) = 0, \\ \frac{\pi_{N+1}(t-1)}{\pi_{N+1}(t-2)} &> \frac{\pi_N(t-1)}{\pi_N(t-2)}, \quad \text{if } t = N+2. \end{aligned}$$

Finally, (31)-(35) along with Lemma 1 establish that

$$\frac{\pi_{N+2}(t)}{\pi_{N+2}(t-1)} > \frac{\pi_{N+1}(t)}{\pi_{N+1}(t-1)}, \quad 3 \leq t \leq N+2.$$

To establish (29), it remains to show that

$$\frac{\pi_{N+2}(2)}{\pi_{N+2}(1)} > \frac{\pi_{N+1}(2)}{\pi_{N+1}(1)} \quad (\text{for } t=2), \quad \text{and} \quad \frac{\pi_{N+2}(1)}{\pi_{N+2}(0)} > \frac{\pi_{N+1}(1)}{\pi_{N+1}(0)} \quad (\text{for } t=1).$$

Given (31) and (32), it is equivalent to showing that, for  $t=2$ ,

$$\frac{\frac{1}{2(1+\beta)}\pi_{N+1}(1) + \frac{1}{2}\pi_{N+1}(2) + \frac{\beta}{2(1+\beta)}\pi_{N+1}(3)}{\frac{1}{2}\pi_{N+1}(0) + \frac{1}{2}\pi_{N+1}(1) + \frac{\beta}{2(1+\beta)}\pi_{N+1}(2)} > \frac{\frac{1}{2(1+\beta)}\pi_N(1) + \frac{1}{2}\pi_N(2) + \frac{\beta}{2(1+\beta)}\pi_N(3)}{\frac{1}{2}\pi_N(0) + \frac{1}{2}\pi_N(1) + \frac{\beta}{2(1+\beta)}\pi_N(2)},$$

and for  $t=1$ ,

$$\frac{\frac{1}{2}\pi_{N+1}(0) + \frac{1}{2}\pi_{N+1}(1) + \frac{\beta}{2(1+\beta)}\pi_{N+1}(2)}{\frac{1}{2}\pi_{N+1}(0) + \frac{\beta}{2(1+\beta)}\pi_{N+1}(1)} > \frac{\frac{1}{2}\pi_N(0) + \frac{1}{2}\pi_N(1) + \frac{\beta}{2(1+\beta)}\pi_N(2)}{\frac{1}{2}\pi_N(0) + \frac{\beta}{2(1+\beta)}\pi_N(1)}.$$

Both inequalities follow from the induction hypothesis and Lemma 2.

**PROOF OF PROPOSITION 3.4:** We prove the proposition by induction on  $N$ . Define  $Y_N(t) \equiv 1 - \bar{p}_N(t)$  so that (20) can be written as

$$(36) \quad \frac{y_{N+1}(t)}{y_{N+1}(t-1)} > \frac{y_N(t)}{y_N(t-1)}, \quad 1 \leq t \leq N.$$

We shall verify in Lemma 1 that (36) holds for  $N = 1, 2, 3$ . Suppose that (36) holds for  $N \geq 3$ . We need to show that (36) holds for  $N+1$ , that is,

$$(37) \quad \frac{y_{N+2}(t)}{y_{N+2}(t-1)} > \frac{y_{N+1}(t)}{y_{N+1}(t-1)}, \quad 1 \leq t \leq N+1.$$

We first use (30) to obtain a recursive relation

$$(38) \quad y_{N+1}(t) = \begin{cases} \frac{1}{2(1+\beta)}y_N(t-1) + \frac{1}{2}y_N(t) + \frac{\beta}{2(1+\beta)}y_N(t+1), & \text{if } t \geq 1, \\ \frac{1}{2(1+\beta)}y_N(0) + \frac{1}{2} + \frac{\beta}{2(1+\beta)}y_N(1), & \text{if } t = 0. \end{cases}$$

Similarly, we obtain the  $y_{N+2}(t)$  term in (37)

$$y_{N+2}(t) = \begin{cases} \frac{1}{2(1+\beta)}y_{N+1}(t-1) + \frac{1}{2}y_{N+1}(t) + \frac{\beta}{2(1+\beta)}y_{N+1}(t+1), & \text{if } t \geq 1, \\ \frac{1}{2(1+\beta)}y_{N+1}(0) + \frac{1}{2} + \frac{\beta}{2(1+\beta)}y_{N+1}(1), & \text{if } t = 0. \end{cases}$$

(39)

We then note that a version of Proposition 1 holds for the modified economy with  $N + 1$  stages, with  $N + 1$  in place of  $N$  everywhere. Using (15) and the induction hypothesis, we obtain

$$(40) \quad \begin{aligned} \frac{y_{N+1}(t+1)}{y_{N+1}(t)} &> \frac{y_N(t+1)}{y_N(t)}, & \frac{y_{N+1}(t)}{y_{N+1}(t-1)} &> \frac{y_N(t)}{y_N(t-1)}, \\ \frac{y_{N+1}(t-1)}{y_{N+1}(t-2)} &> \frac{y_N(t-1)}{y_N(t-2)}, & \text{if } 2 \leq t \leq N-1; \end{aligned}$$

$$(41) \quad \begin{aligned} y_{N+1}(t+1) = y_N(t+1) = y_N(t) = 0, & \quad \frac{y_{N+1}(t)}{y_{N+1}(t-1)} > \frac{y_N(t)}{y_N(t-1)}, \\ \frac{y_{N+1}(t-1)}{y_{N+1}(t-2)} > \frac{y_N(t-1)}{y_N(t-2)}, & \quad \text{if } t = N; \end{aligned}$$

$$(42) \quad \begin{aligned} y_{N+1}(t+1) = y_{N+1}(t) = y_N(t+1) = y_N(t) = y_N(t-1) = 0, \\ \frac{y_{N+1}(t-1)}{y_{N+1}(t-2)} > \frac{y_N(t-1)}{y_N(t-2)}, & \quad \text{if } t = N+1. \end{aligned}$$

Finally, (38)-(42) and Lemma 1 imply that

$$\frac{y_{N+2}(t)}{y_{N+2}(t-1)} > \frac{y_{N+1}(t)}{y_{N+1}(t-1)}, \quad 2 \leq t \leq N+1.$$

To establish (37), it thus remains to show that

$$\frac{y_{N+2}(1)}{y_{N+2}(0)} > \frac{y_{N+1}(1)}{y_{N+1}(0)},$$

which, given (38) and (39), is equivalent to showing that

$$(43) \quad \frac{\frac{1}{2(1+\beta)}y_{N+1}(0) + \frac{1}{2}y_{N+1}(1) + \frac{\beta}{2(1+\beta)}y_{N+1}(2)}{\frac{1}{2(1+\beta)}y_{N+1}(0) + \frac{1}{2} + \frac{\beta}{2(1+\beta)}y_{N+1}(1)} > \frac{\frac{1}{2(1+\beta)}y_N(0) + \frac{1}{2}y_N(1) + \frac{\beta}{2(1+\beta)}y_N(2)}{\frac{1}{2(1+\beta)}y_N(0) + \frac{1}{2} + \frac{\beta}{2(1+\beta)}y_N(1)}.$$

To establish (43), by Lemma 1, it suffices to show that

$$\frac{y_{N+1}(2)}{y_{N+1}(1)} > \frac{y_N(2)}{y_N(1)}, \quad \frac{y_{N+1}(1)}{1} > \frac{y_N(1)}{1}, \quad \frac{y_{N+1}(0)}{y_{N+1}(0)} = \frac{y_N(0)}{y_N(0)}.$$

The first inequality follows from the induction hypothesis, the second follows from (18) for index  $N$  in an economy with  $N + 1$  stages, and the last equality is trivial. This establishes (37), and thus completes the proof.

**PROOF OF PROPOSITION 3.5:** In light of (10), (12) and (13), it suffices to show that, for each  $t \geq 0$ ,

$$(44) \quad \lim_{N \rightarrow \infty} p_N(t) = 0.$$

We proceed by first showing that the limit exists. Similarly as in the proofs of Propositions 3.1 and 3.2, we can show that  $p_N(t)$  is monotonically decreasing in  $N$ . The recursive relations in (24) imply that, for all  $N \geq 2$ ,  $p_N(t)$  is a weighted average of the first-stage prices  $p_1(-1)$ ,  $p_1(0)$ ,  $\dots$ , and  $p_1(t+N-1)$ . This together with (14) and the fact that  $p_1(-1) = 0$  implies that  $p_N(t)$  is uniformly bounded from below by 0 and from above by 1. Therefore, for each  $t \geq -1$ , the limit of  $p_N(t)$  as  $N \rightarrow \infty$  exists. Denote this limit by  $p(t)$ . Then, trivially  $p(-1) = 0$ , and

$$(45) \quad 0 \leq p(t) \leq 1,$$

for each  $t \geq 0$ . It remains to show that  $p(t) = 0$  for each  $t \geq 0$ . For convenience, we rewrite here (24) for index  $N$  and for each  $t \geq 0$ :

$$p_N(t) = \frac{1}{2(1+\beta)} p_{N-1}(t-1) + \frac{1}{2} p_{N-1}(t) + \frac{\beta}{2(1+\beta)} p_{N-1}(t+1).$$

Since each of the four terms in the above equation converges to a finite limit, taking  $N \rightarrow \infty$  on both sides of the equation leads to

$$p(t) = \frac{1}{2(1+\beta)} p(t-1) + \frac{1}{2} p(t) + \frac{\beta}{2(1+\beta)} p(t+1),$$

which can be rewritten as  $p(t+1) - p(t) = [p(t) - p(t-1)]/\beta$ . By iterating on  $t$ , we get

$$(46) \quad p(t+1) - p(t) = \left(\frac{1}{\beta}\right)^{t+1} [p(0) - p(-1)].$$

Summing up both sides of (46) through periods  $0, \dots, t$ , and using  $p(-1) = 0$  and  $0 < \beta < 1$ , we have  $p(t+1) = p(0)[(1/\beta)^{t+2} - 1]/[(1/\beta) - 1]$ .<sup>7</sup> It follows that, for each  $t \geq 0$ ,

$$(47) \quad p(t) = \left[ \frac{(1/\beta)^{t+1} - 1}{(1/\beta) - 1} \right] p(0).$$

Equation (47) implies that  $p(0) = 0$ . If otherwise  $p(0) > 0$ , then there exists some  $\tau \geq 0$  such that  $p(t) > 1$  for  $t \geq \tau$ , a contradiction to (45). It follows immediately that  $p(t) = 0$  for  $t \geq 0$ . This completes the proof.

LEMMA 1: *Let  $A, B, C, D$  and  $a, b, c, d$  be arbitrary nonnegative real numbers. Then,*

$$(48) \quad \frac{\frac{1}{2(1+\beta)}B + \frac{1}{2}C + \frac{\beta}{2(1+\beta)}D}{\frac{1}{2(1+\beta)}A + \frac{1}{2}B + \frac{\beta}{2(1+\beta)}C} > \frac{\frac{1}{2(1+\beta)}b + \frac{1}{2}c + \frac{\beta}{2(1+\beta)}d}{\frac{1}{2(1+\beta)}a + \frac{1}{2}b + \frac{\beta}{2(1+\beta)}c}$$

if one of the following three conditions holds:

- (i)  $\frac{D}{C} \geq \frac{d}{c}$ ,  $\frac{C}{B} \geq \frac{c}{b}$ ,  $\frac{B}{A} \geq \frac{b}{a}$ , with at least one strict inequality,
- (ii)  $D = d = c = 0$ ,  $\frac{C}{B} \geq \frac{c}{b}$ ,  $\frac{B}{A} \geq \frac{b}{a}$ , with at least one strict inequality,
- (iii)  $C = D = b = c = d = 0$ ,  $\frac{B}{A} > \frac{b}{a}$ ,

where all variables are strictly positive unless specified otherwise.

PROOF: We first prove (48) under (i). Cross multiplying the terms on both sides of (48) and expanding the resulting expressions show that (48) is equivalent to the following inequality:

$$\begin{aligned} & \frac{1}{4(1+\beta)^2}Aa + \frac{1}{4(1+\beta)}Bb + \frac{\beta}{4(1+\beta)^2}Bc + \frac{1}{4(1+\beta)}Ca + \frac{1}{4}Cb + \frac{\beta}{4(1+\beta)}Cc \\ & + \frac{\beta}{4(1+\beta)^2}Da + \frac{\beta}{4(1+\beta)}Db + \frac{\beta^2}{4(1+\beta)^2}Dc \\ & > \frac{1}{4(1+\beta)^2}Ab + \frac{1}{4(1+\beta)}Ac + \frac{\beta}{4(1+\beta)^2}Ad + \frac{1}{4(1+\beta)}Bb + \frac{1}{4}Bc + \frac{\beta}{4(1+\beta)}Bd \\ & + \frac{\beta}{4(1+\beta)^2}Cb + \frac{\beta}{4(1+\beta)}Cc + \frac{\beta^2}{4(1+\beta)^2}Cd \end{aligned}$$

Using (i) to compare the two sides of (49) term by term leads to a conclusion that the terms on the left-hand side are always larger than or equal to the corresponding terms on the right-hand side, except for those terms involving  $Bc$  and  $Cb$ . We thus need to show that

$$\frac{\beta}{4(1+\beta)^2}Bc + \frac{1}{4}Cb \geq \frac{1}{4}Bc + \frac{\beta}{4(1+\beta)^2}Cb,$$

or, by collecting terms, that

$$\frac{1}{4} \left[ 1 - \frac{\beta}{(1+\beta)^2} \right] (Bc - Cb) \leq 0.$$

The above inequality holds since  $0 < \beta < 1$  and  $Bc \leq Cb$  by (i). Since there is at least one strict inequality in (i), (49) holds, and so does (48). The proof of (48) under (ii) or (iii) is similar, with the specified zero terms imposed in (49). This completes the proof.

LEMMA 2: For any positive real numbers  $A, B, C, D$  and  $a, b, c, d$ , the conditions  $\frac{B}{A} > \frac{b}{a}$ ,  $\frac{C}{B} > \frac{c}{b}$ , and  $\frac{D}{C} > \frac{d}{c}$  imply that

$$\frac{\frac{1}{2(1+\beta)}B + \frac{1}{2}C + \frac{\beta}{2(1+\beta)}D}{\frac{1}{2}A + \frac{1}{2}B + \frac{\beta}{2(1+\beta)}C} > \frac{\frac{1}{2(1+\beta)}b + \frac{1}{2}c + \frac{\beta}{2(1+\beta)}d}{\frac{1}{2}a + \frac{1}{2}b + \frac{\beta}{2(1+\beta)}c},$$

and

$$\frac{\frac{1}{2(1+\beta)}B + \frac{\beta}{2(1+\beta)}C}{\frac{1}{2}A + \frac{\beta}{2(1+\beta)}B} > \frac{\frac{1}{2(1+\beta)}b + \frac{\beta}{2(1+\beta)}c}{\frac{1}{2}a + \frac{\beta}{2(1+\beta)}b}.$$

PROOF: (Similar to the proof of Lemma 1).

LEMMA 3: In the perfect foresight equilibrium, the inequalities

$$\frac{1 - \bar{p}_{N+1}(t)}{1 - \bar{p}_{N+1}(t-1)} > \frac{1 - \bar{p}_N(t)}{1 - \bar{p}_N(t-1)}, \quad 1 \leq t \leq N,$$

hold for  $N = 1, 2, 3$ .

PROOF: Equations (12), (14), and (15) together with  $p_1(-1) = 0$  imply that

$$\bar{p}_1(0) = \frac{1}{2}, \quad \bar{p}_1(t) = 1, \quad t \geq 1.$$

Using the above solutions and repeatedly applying (30) result in the following solutions:

$$\bar{p}_2(0) = \frac{1 + 2\beta}{4(1 + \beta)}, \quad \bar{p}_2(1) = \frac{3 + 4\beta}{4(1 + \beta)}, \quad \bar{p}_2(t) = 1, \quad t \geq 2,$$

$$\bar{p}_3(0) = \frac{1 + 4\beta}{8(1 + \beta)}, \quad \bar{p}_3(1) = \frac{4 + 13\beta + 8\beta^2}{8(1 + \beta)^2},$$

$$\bar{p}_3(2) = \frac{7 + 16\beta + 8\beta^2}{8(1 + \beta)^2}, \quad \bar{p}_3(t) = 1, \quad t \geq 3,$$

$$\bar{p}_4(0) = \frac{1 + 9\beta + 17\beta^2 + 8\beta^3}{16(1 + \beta)^3}, \quad \bar{p}_4(1) = \frac{5 + 29\beta + 41\beta^2 + 16\beta^3}{16(1 + \beta)^3},$$

$$\bar{p}_4(2) = \frac{11 + 44\beta + 48\beta^2 + 16\beta^3}{16(1 + \beta)^3}, \quad \bar{p}_4(3) = \frac{15 + 48\beta + 48\beta^2 + 16\beta^3}{16(1 + \beta)^3},$$
$$\bar{p}_4(t) = 1, \quad t \geq 4.$$

It is then straightforward to verify the claimed inequality by direct substitutions.

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## NOTES

1. Therefore the multi-stage production structure creates a “real rigidity” in the sense of Ball and Romer (1990). It is also important to recognize that, the meaning of “inflation persistence” here is different from that in, for example, Fuhrer and Moore (1995). By “inflation persistence,” they mean that disinflation causes an output loss (see also Ball (1994, 1995)), while here we refer to the high serial correlation in the response of inflation to a monetary shock (see also Nelson (1998)).
2. Under a state-dependent pricing rule, our results will hold as long as firms at *some* stages of production face heterogeneous menu costs. Casual observations suggest that many firms do face different costs of price adjustment.
3. In our closed-economy model with no capital or government spending, real GDP corresponds to aggregate consumption.
4. We derive the equilibrium conditions and report the log-linearization process in a Technical Appendix, which is available upon request.
5. To see why this measure corresponds to the first order auto-correlation, consider an arbitrary AR(1) process  $x(t) = \rho x(t - 1) + e(t)$ , where, under perfect foresight, the residual term  $e(t) = 0$ . Thus, the ratio  $x(t)/x(t - 1)$  measures the magnitude of persistence.
6. For a quantitative model of nominal wage rigidity with micro-foundations, see, for example, Huang and Liu (1998).
7. Proposition 4 in fact holds even in the case without discounting, i.e., with  $\beta = 1$ . To see this, note that in this case (46) implies that  $p(t) = tp(0)$  for all  $t \geq 1$ . Therefore, the only value that  $p(0)$  can take is 0. If otherwise  $p(0) > 0$ , then  $p(t) > 1$  for all  $t \geq 1/p(0)$ , a contradiction to (45). It then follows immediately that  $p(t) = 0$  for all  $t \geq 0$ .

TABLE I  
RESPONSE OF PRICE INDICES

$\bar{p}_n(t)$	$n = 1$	$n = 2$	$n = 5$	$n = 10$	$n = 20$
$\bar{p}_n(0)$	0.50	0.37	0.24	0.17	0.12
$\bar{p}_n(1)$	1.00	0.87	0.65	0.49	0.36
$\bar{p}_n(2)$	1.00	1.00	0.89	0.73	0.56
$\bar{p}_n(3)$	1.00	1.00	0.98	0.88	0.72

TABLE II  
OUTPUT PERSISTENCE

	$N = 1$	$N = 2$	$N = 5$	$N = 10$	$N = 20$
Contract Multiplier	0.00	0.20	0.46	0.62	0.73
Half Life	0.50	0.63	0.93	1.41	2.01

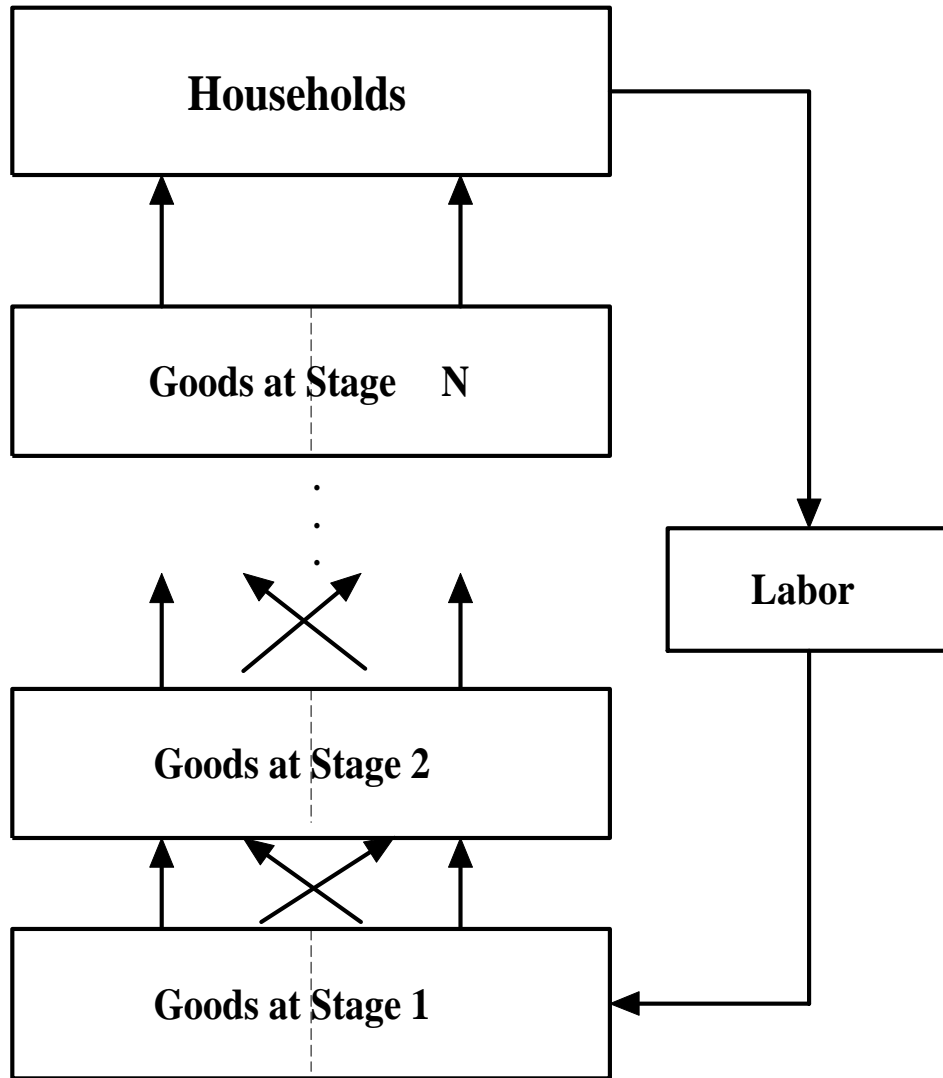


Figure I.—Chain structure of the economy

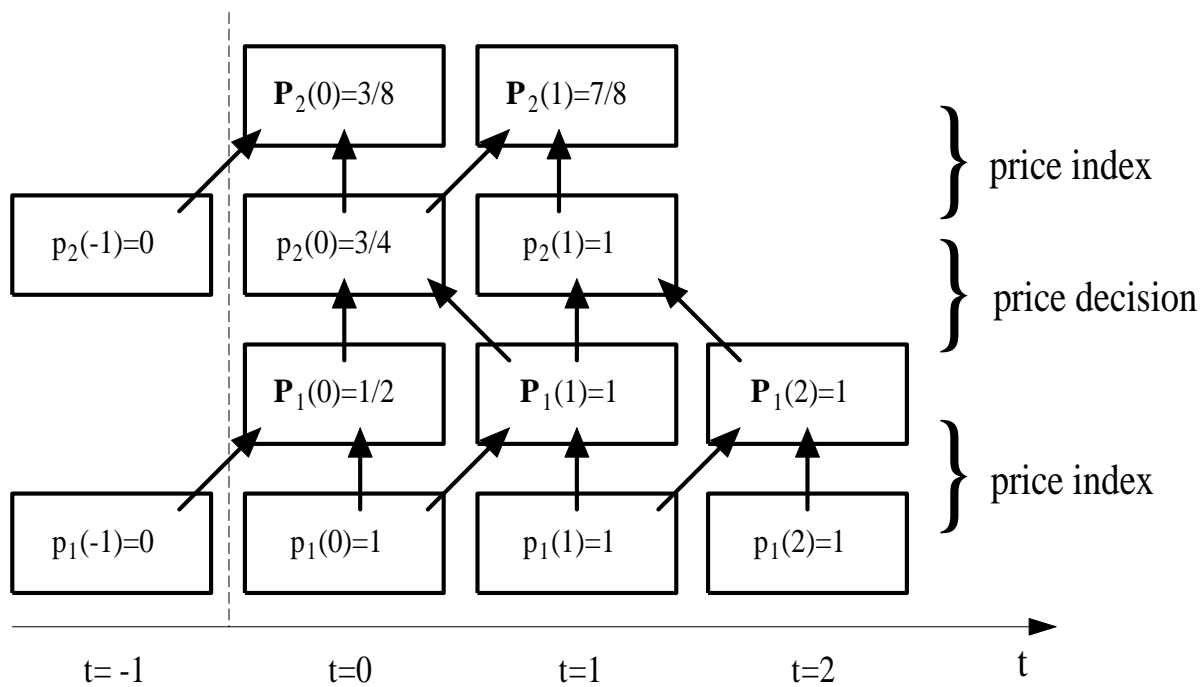


Figure II.—Price adjustments across stages ( $N = 2, \beta = 1$ )