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Staggered Contracts and Business Cycle Persistence *

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Abstract:
Staggered price and staggered wage contracts are commonly viewed as similar mechanisms in generating persistent real effects of monetary shocks. In this paper, we distinguish the two mechanisms in a dynamic stochastic general equilibrium framework. We show that, although the dynamic price setting and wage setting equations are alike, a key parameter governing persistence is linked to the underlying preferences and technologies in different ways. Under the staggered wage mechanism, an intertemporal smoothing incentive in labor supply creates a real rigidity that is absent under the staggered price mechanism. Consequently, the two mechanisms have different implications on persistence. While the staggered price mechanism by itself does not contribute to, the staggered wage mechanism plays an important role in generating persistence.

Keywords:
Staggered Contracts; Business Cycle Persistence; Monetary Policy

JEL classification: E24, E32, E52

1 Introduction

How monetary policy shocks affect business cycle duration has been a challenging issue concerning economists and policy makers. Recent empirical studies such as Christiano, Eichenbaum, and Evans (1999) reveal that monetary shocks can have long-lasting effects on real activities. Yet, it has been a difficult task to identify monetary transmission mechanisms that can contribute to generating such effects.[1]

In a seminal paper, Taylor (1980) proposes a staggered wage mechanism to help solve this persistence issue. In his model, nominal wages are set in a staggered fashion. That is, not all wage decisions are made at the same time, and each wage, after being set, is fixed for a short period of time such as a year. As summarized in Taylor (1999), there is much empirical evidence that price contracts and wage contracts are staggered. Taylor (1980) shows that this staggered wage mechanism can lead to endogenous wage inertia and thereby persistence in employment movements following a temporary shock. He states the intuition as follows:

> Because of the staggering, some firms will have established their wage rates prior to the current negotiations, but others will establish their wage rates in future periods. Hence, when considering relative wages, firms and unions must look both forward and backward in time to see what other workers will be paid during their own contract period. In effect, each contract is written relative to other contracts, and this causes shocks to be passed on from one contract to another . . . contract formation in this model generates an inertia of wages which parallels the persistence of unemployment.

More recently, Chari, Kehoe, and McGrattan (CKM) (1998) carry this intuition to a general equilibrium environment. But, perhaps surprisingly, they find that a staggered *price* mechanism by itself cannot generate persistent real effects following monetary shocks, an apparent puzzle in light of Taylor's insights. There are two interpretations of this puzzle. On one hand, CKM (1998) suggest that it is difficult to explain persistence based on staggered nominal contracts in a general equilibrium framework, and "we should look elsewhere for mechanisms to generate persistence." On the other hand, Taylor (1999) conjectures that, "the findings of Chari, Kehoe, and McGrattan (1998) may indicate that the monopolistic competition (stationary market power) model may not be sufficient as a microeconomic foundation." Behind the two arguments seems to be a common perception that a staggered price mechanism and a staggered wage mechanism are embodied with the same implications on persistence: either that they both contribute to generating persistence or that neither does so.[2]

The purpose of this paper is to suggest a third interpretation of the persistence puzzle. We find that a general equilibrium model along the line of CKM (1998), incorporating staggered wage contracts rather than staggered price contracts, is able to generate substantial persistence. Thus, staggered wage contracts are an important contributing mechanism in generating persistent real effects of monetary shocks, even when the underlying wage setting rule is derived from the standard monopolistic competition framework. The two models have different implications on persistence because, in a general equilibrium environment, the key parameter that governs persistence in the dynamic price setting and the dynamic wage setting equations is a function of the underlying preferences and technologies of the economy. Although the two equations are apparently identical, this functional form and thereby the value of the persistence parameter differ across the two mechanisms.

To facilitate the comparison of the two mechanisms, we construct two models in a symmetric way. The first model features perfectly competitive goods markets, monopolistically competitive labor markets, and households endowed with differentiated labor skills setting nominal wages. The second model, on the other hand, features perfectly competitive labor markets, monopolistically competitive goods markets, and firms producing differentiated goods setting prices. In the spirit of Taylor (1980), we assume that wages and prices are set in a staggered fashion.[3] Following the lead of CKM (1998), we derive the wage setting and the price setting rules from households' and firms' optimizing decisions and thus link these decision rules to the underlying preferences and technologies in the models. We show that a critical parameter governing persistence is the elasticity of relative wage (or price) with respect to aggregate demand in the wage (or price) equation. A greater value of this parameter corresponds to less persistence, because it implies a larger response of wage (or price) decisions to aggregate demand shocks, and thus a faster adjustment of wage (or price) index and a quicker return of aggregate output to steady state. Under the staggered wage mechanism, the value of this parameter is necessarily *less* than one, and decreases with both the elasticity of substitution among differentiated labor skills in the production technology and the degree of relative risk aversion in labor hours in households' preferences. In contrast, the value of this parameter under the staggered price mechanism is necessarily *greater* than one, and increases with the degree of relative risk aversion in labor hours. Consequently, a staggered wage mechanism tends to generate persistence but a staggered price mechanism does not.

To understand the driving forces of these results, we compare the optimal responses of households and firms to a monetary shock in the two models. In the staggered wage model, imperfectly competitive households choose nominal wages to balance the expected marginal utility of leisure and of wage income during their contract periods, taking into account the effects of the wage decisions on the demand for their labor services and thus their wage incomes as well. When an expansionary monetary shock occurs, wage index does not increase proportionally due to staggering in wage setting. Price level does not fully rise either since profit maximization requires that price equal marginal cost determined by wage index. Therefore, real aggregate demand increases, raising both households' income and firms' demand for labor services. The higher income reduces the households' marginal utility of income and the higher labor demand raises their marginal utility of leisure. Utility maximization requires that households who can renew contracts raise wages to rebalance their marginal utility of income and of leisure. We find that the optimal percentage increase in relative wages is necessarily less than the percentage increase in aggregate demand. The reason is that a higher relative wage reduces both the demand for the corresponding type of labor services (substitution effect) and the associated wage income (income effect). These effects both serve to restore the balance between the marginal utility of income and of leisure. Thus the optimal increase in relative wages is small. Consequently, wage index rises slowly, and movements in aggregate output and employment, after their initial responses to the shock, are also slow and persistent. Moreover, the easier to substitute across labor skills and the more willing the households to smooth labor hours, the smaller the optimal wage adjustment, and thus the larger the output persistence. If we measure the magnitude of persistence by the ratio of output response at the end of the initial contract duration to that in the impact period (i.e., a "contract multiplier"), this ratio is about 40% under our calibrated parameter values.

The staggered price mechanism works differently. Under this mechanism, imperfectly competitive firms choose prices to maximize expected profits during their contract periods, taking into account the effects of the price decisions on the demand for their goods and thus their revenues as well. We show that the optimal price is a linear function of a firm's expected marginal costs during its contract periods. Thus a higher price will be set if the firm is expecting higher marginal costs. Staggered price setting allows an expansionary monetary shock to raise real aggregate demand and thus firms' demand for labor services. On the other hand, following the shock, households receive more real income, and consequently they are willing to work less at each given real wage. The outward shift of labor demand curve and the inward shift of labor supply curve both serve to drive up real wage and thus the real marginal cost of production. If households prefer smoothed labor hours, the equilibrium percentage increase in real wage will exceed the increase in aggregate demand, causing marginal cost to rise by more than aggregate demand does. In response, profit-maximizing firms will fully adjust their prices whenever they have the chance to renew contracts. Consequently, movements in aggregate output and employment, after their initial responses to the shock, are fast and transitory. In contrast to the staggered wage model, the contract multiplier is here negative for reasonable parameter values.

In the literature, there are three strands of research work that are related to ours. The first strand is the staggered price contract literature centering on the CKM (1998) persistence puzzle. For example, Bergin and Feenstra (1998) show that adding a non-CES production function and factor specificity to a staggered price model can contribute to generating persistence; Kiley (1997) demonstrates that assuming a high degree of increasing returns at individual firm level can help produce persistence in a staggered price model; and Gust (1997) emphasizes the importance of constraining factor mobility across sectors. The second strand of literate related to our work is the state-dependent pricing literature. Dotsey, King, and Wolman (1999) provide a general equilibrium framework for analyzing the implications of state-dependent price setting rules. Dotsey, et. al (1997) show that staggered price setting can arise from small menu costs, and incorporating variable capacity utilization in such a model is a promising mechanism in delivering persistence. The third strand is the nominal wage contract literature. Following the seminal work of Blanchard and Kiyotaki (1987) and Blanchard (1986), attempts have been made to model staggered wage contracts in a dynamic general equilibrium environment. For example, Erceg (1997) analyzes a model with both staggered price and staggered wage contracts and studies the role of this double staggering mechanism in propagating monetary shocks, while Huang and Liu (1999) show that adding a staggered price mechanism on top of a staggered wage mechanism does not help magnify persistence. The recent work by Cho, Cooley, and Phaneuf (1997) evaluates the welfare effect of nominal wage contracts. In summary, there has been a renewed interest in identifying monetary propagation mechanisms within the framework of staggered nominal contracts. Yet, little has been done to explore the microstructures that may distinguish the staggered wage mechanism from the staggered price mechanism. In this paper, we distinguish the two mechanisms in their capabilities of generating persistence. It is important to emphasize that we do not attempt to propose a single friction model that is able to fully account for the dynamic output responses to monetary shocks. In fact, the recent work by Christiano, et. al (1997) suggests that it is unlikely for a single-friction model to provide a complete account of the real effects of monetary shocks. To provide such an account, a combination of frictions is required. Our work suggests that, in such a multi-friction model, staggered wage contracts can be an important contributing mechanism.

The rest of the paper is organized as follows. Section 2 illustrates Taylor's (1980) original intuition and briefly describes the CKM (1998) persistence puzzle.

Section 3 and 4 present two general equilibrium models with staggered wage and with staggered price contracts, respectively, and use analytical solutions to distinguish the two mechanisms in their potentials of generating persistence. Section 5 evaluates the quantitative implications of the two mechanisms based on two calibrated models with capital. Section 6 concludes the paper. The models with capital are described in the Appendix.

2 Taylor's Insights and the CKM Persistence Puzzle

In this section, we use a simplified version of Taylor's (1980) model to illustrate his original intuition. We then describe the CKM (1998) persistence puzzle to motivate our present work.

2.1 A Simple Model in the Spirit of Taylor (1980)

Consider an economy in which, as in Taylor (1980), prices are set for N periods and remain fixed during these "contract periods," where $N > 1$. In each period, a fraction $1/N$ of firms can set prices, and in doing so, they take into account the prevailing price which, at any point of time, is an average of the N contractual prices determined in the current and the previous $N-1$ periods. Therefore, when setting new prices, firms look at both the future and the past price decisions because these are part of the prevailing price. When $N = 2$, the price setting rule is fully described by the following equations:

$$p_t = \frac{1}{2}(x_t + x_{t-1}), \tag{1}$$

$$x_t = \frac{1}{2}(p_t + \mathrm{E}_t p_{t+1}) + \frac{\gamma}{2}(y_t + \mathrm{E}_t y_{t+1}) + e_t, \tag{2}$$

where x_t denotes the price decision, p_t the prevailing price level, y_t the aggregate output, and E_t is a conditional expectation operator. All variables are in log-terms, and e_t is a shock to price setting. The parameter γ measures the responsiveness of price decisions to changes in aggregate demand conditions. The system can be closed by assuming a money demand equation $y_t = m_t - p_t$. To focus on monetary shocks, we set $e_t = 0$. The model can then be reduced to a second order difference equation in x_t by substituting for p_t and y_t using (1) and the money demand equation, respectively. Under an additional assumption that the money stock m_t follows a random walk process, a simple solution to this difference equation can be obtained, and the implied dynamic output equation is given by

$$y_t = a y_{t-1} + \frac{1+a}{2}(m_t - m_{t-1}), \quad \text{where} \quad a = \frac{1-\sqrt{\gamma}}{1+\sqrt{\gamma}}. \tag{3}$$

Two special cases are worth mentioning: if $\gamma = 1$, then $a = 0$ and there is no persistence; if $\gamma = 0$, then $a = 1$ and the output follows a random walk process. In general, a smaller γ corresponds to a larger a and hence more output persistence. Taylor (1980, 1999) notes that the autoregressive output process arises from the staggering in price setting. Therefore, a model with staggered price contracts can potentially generate large amount of persistence, provided that the key parameter γ is small.

In Taylor's (1980) original setup, γ is a structural parameter void of any distinctions between price setting and wage setting, and the above arguments apply to both mechanisms with the corresponding notations being appropriately interpreted.

2.2 The CKM (1998) Persistence Puzzle

CKM (1998) carry Taylor's (1980) intuition to a general equilibrium business cycle model with staggered *price* contracts, and thereby link the parameter γ to underlying economic fundamentals such as preferences and technologies. However, they find that there is no persistence in output dynamics because the magnitude of γ so determined is too large for empirically plausible values of preference and technology parameters. CKM (1998) test the sensitivity of this result by including various features such as convex demand curve, specific factor of production, and zero-income-effect utility function, and find the result robust.

There are two different interpretations of this persistence puzzle. On one side, it is inferred that staggered nominal contracts may not be an important contributing mechanism in generating persistence in a general equilibrium setup. On the other side, it is conjectured that the conventional monopolistic competition framework may not be adequate for deriving the price setting equation. This puzzle has stimulated much research in combining various other mechanisms with the staggered price mechanism to lower the value of γ.

In this paper, we reassess the persistence puzzle from a different perspective. We realize that, with optimizing individuals, a staggered *wage* mechanism, after all, may be quite different from a staggered *price* mechanism in generating persistence. Our finding is that such a difference does exist because the parameter γ is determined by different economic forces in models with the two mechanisms. Such fine distinctions cannot possibly be uncovered unless the optimizing behaviors of households and firms are explicitly modeled.

3 A Model with Staggered Wage Contracts

In this section, we describe a general equilibrium model with staggered wage contracts. In the model economy, there is a large number of infinitely-lived households who are endowed with differentiated labor skills indexed in the interval $[0,1]$, and there is a large number of *identical* firms who use all types of the labor services to produce a homogeneous consumption good. In each period t, the economy experiences a realization of shocks s_t, while the history of events up to date t is $s^t \equiv (s_0, \cdots, s_t)$ with probability $\pi(s^t)$. The initial realization s_0 is given.

Production technology is given by $Y(s^t) = L(s^t)$, where $L(s^t) = \left[\int_0^1 L(i,s^t)^{\frac{\sigma-1}{\sigma}} di\right]^{\frac{\sigma}{\sigma-1}}$ is a Dixit-Stiglitz (1970) type of composite of labor services. In the production function, $L(i,s^t)$ is the labor service provided by household i at s^t, and σ is the elasticity of substitution among different types of labor services, where $\sigma > 1$.

Firms behave competitively. Upon the realization of s^t, they choose output $Y(s^t)$ and labor services $\{L(i,s^t)\}_{i\in(0,1)}$ to maximize profit $P(s^t)Y(s^t) - \int_0^1 W(i,s^t)L(i,s^t)di$, subject to the production technology, taking price $P(s^t)$ and wages $\{W(i,s^t)\}_{i\in(0,1)}$ as given. The resulting demand function for the labor service of type i is

$$L^d(i,s^t) = \left[\frac{W(i,s^t)}{\bar{W}(s^t)}\right]^{-\sigma} L(s^t), \qquad (4)$$

where $\bar{W}(s^t) = \left[\int_0^1 W(i,s^t)^{1-\sigma}\, di\right]^{1/(1-\sigma)}$ is a wage index. Zero-profit condition implies that $P(s^t) = \bar{W}(s^t)$.

Households are price-takers in goods markets and monopolistic competitors in labor markets. They take the labor demand schedule (4) as given and set wages in a staggered fashion. In particular, in each period t, there is a fraction $1/N$ of households that can set new wages upon the realization of s^t. Once a wage is set, it has to remain fixed for N periods. We sort the indices of households so that those indexed by $i \in [0, 1/N]$ set wages in periods $t, t+N, t+2N, \cdots$, those indexed by $i \in (1/N, 2/N]$ set wages in periods $t+1, t+N+1, t+2N+1, \cdots$, and so on.

Household i has a utility function

$$U^i \equiv \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) \{\log(C^\star(i,s^t)) + V(L(i,s^t))\},$$

where $C^\star(i) = [bC(i)^\nu + (1-b)(M(i)/P)^\nu]^{1/\nu}$ is a CES composite of consumption and real money balances, and $V(\cdot)$ is a strictly decreasing and strictly concave function. Upon the realization of s^t, the household solves its utility maximization problem by choosing consumption $C(i,s^t)$, nominal money balances $M(i,s^t)$, and one-period nominal bond holdings $B(i,s^{t+1})$, taking prices $P(s^t)$ and $D(s^{t+1}|s^t)$ as given. If the household is a member of the cohort that can set new wages, it also chooses a nominal wage $W(i,s^t)$ for the current and the next $N-1$ periods, taking the labor demand schedule (4) as given. The utility maximization is subject to a sequence of budget constraints

$$P(s^t)C(i,s^t) + \sum_{s^{t+1}} D(s^{t+1}|s^t)B(i,s^{t+1}) + M(i,s^t) \leq \\ W(i,s^t)L^d(i,s^t) + \Pi(i,s^t) + B(i,s^t) + M(i,s^{t-1}) + T(i,s^t),$$

and a borrowing constraint $B(i,s^t) \geq -\bar{B}$ for some large positive number $\bar{B}$, for each s^t, with initial conditions $M(i,s^{-1})$ and $B(i,s^0)$ given. Here $B(i,s^{t+1})$ is a one-period nominal bond that costs $D(s^{t+1}|s^t)$ dollars in s^t and pays off one dollar in the next period if s^{t+1} is realized, $\Pi(i,s^t)$ is the household's claim to firms' profits, and $T(i,s^t)$ is a nominal transfer to the household.

To close the description of the model, we need to specify a monetary policy. We assume that newly created money is equally distributed to all households via lump-sum transfers so that $\int_0^1 T(i,s^t)\,di = M(s^t) - M(s^{t-1})$.

An equilibrium in this economy consists of a set of allocations $C(i,s^t)$, $M(i,s^t)$, $B(i,s^{t+1})$ for each household i, and $Y(s^t)$ and $\{L(i,s^t)\}_{i\in[0,1]}$ for firms, together with prices $D(s^{t+1}|s^t)$, $P(s^t)$, $\bar{W}(s^t)$, and $\{W(i,s^t)\}_{i\in[0,1]}$ that satisfy the following conditions: (i) taking prices as given, firms' allocations solve their profit maximization problem; (ii) taking prices and all wages but its own as given, each household's allocations and wage solve its utility maximization problem; (iii) goods market, money market, and bond market clear; and (iv) monetary policy is as specified.

In what follows, we focus on a symmetric equilibrium in which all households in a given cohort make identical wage decisions. Since there are complete contingent bond markets and consumption and leisure are additively-separable in the utility function, equilibrium consumption flows and real money balances are identical across all households.[4] Combining this observation with the market clearing conditions, we have $C(i,s^t) = C(s^t) = Y(s^t)$ and $M(i,s^t) = M(s^t)$ for all i. To help exposition, we impose a static money demand function $P(s^t)Y(s^t) = M(s^t)$ for now and relax this assumption in Section 5.

To see how the staggered wage mechanism can help generate persistence, we consider first the case with no staggering, that is, with $N = 1$. The first order condition with respect to household i's wage decision implies that

$$\frac{W(i,s^t)}{P(s^t)} = \frac{\sigma}{\sigma-1}\frac{-V_l(i,s^t)}{U_c(i,s^t)}, \tag{5}$$

where $-V_l(i,s^t)$ and $U_c(i,s^t)$ are the household's marginal utility of leisure and of consumption, respectively. Equation (5) says that the optimal real wage (or relative wage since $P = \bar{W}$ in equilibrium) is a constant "markup" over the marginal rate of substitution between leisure and consumption. When the marginal utility of leisure rises, the household increases its wage to reduce the demand for its labor services; when the marginal utility of consumption increases, the household lowers its wage to raise its labor income and consumption.[5] With $N = 1$, all households make identical wage decisions in a symmetric equilibrium so that $W(i,s^t) = \bar{W}(s^t) = P(s^t)$ and $L(i,s^t) = L(s^t)$. The real wage is thus always constant and a monetary shock only results in a proportionally higher price level, leaving all real variables unchanged.

In the case with staggered wage decisions, i.e., with $N > 1$, however, the situation is different. As a cohort of households makes wage decisions, the rest $N-1$ cohorts cannot set new wages. Thus, when a household raises its wage, it also raises its relative wage, resulting in a lower demand for its labor services and a lower wage income given that $\sigma > 1$. Before turning to the N-period wage setting rule, we develop first a quantitative measure of the contemporaneous response of relative wage to a given aggregate demand shock, assuming that each household takes wage index as given in making wage decisions and there is no forward- or backward-looking effects. These assumptions are to be relaxed later. Notice that (5) can be rewritten as

$$\frac{W(i,s^t)}{\bar{W}(s^t)} = \frac{\sigma}{b(\sigma-1)}\left\{-V_l\left[\left(\frac{W(i,s^t)}{\bar{W}(s^t)}\right)^{-\sigma} Y(s^t)\right]\right\} Y(s^t), \tag{6}$$

where we have used the zero-profit condition $P(s^t) = \bar{W}(s^t)$, the labor demand equation (4), the money demand equation $P(s^t)Y(s^t) = M(s^t)$, and the market clearing condition $C(i,s^t) = Y(s^t) = L(s^t)$ for all i.

Suppose that there is now an expansionary monetary shock. Since the wage index does not rise proportionally due to the staggering, the real aggregate demand rises. If household i's relative wage remained constant, the demand for its labor services $L^d(i,s^t)$ and thus its marginal utility of leisure would rise. Utility maximization requires that the household raise its wage to maintain (6). The equilibrium relative wage is a fixed point of the function $f(x,Y) \equiv \frac{\sigma}{b(\sigma-1)}\{-V_l[x^{-\sigma}Y]\}Y$

with respect to $x \equiv W/\bar{W}$. To see how much the relative wage has to be raised in response to a given demand shock, we take total differentiation of (6) to obtain the elasticity of the relative wage with respect to the aggregate output

$$\epsilon_{x,Y} \equiv \frac{dx}{dY}\frac{Y}{x} = \frac{1+\xi}{1+\sigma\xi}, \tag{7}$$

where σ is the elasticity of substitution among different types of labor services, and $\xi \equiv V_{ll}L(i)/V_l$ measures the household's relative risk aversion in labor hours. Given that $\sigma > 1$ and $\xi > 0$, two observations are worth mentioning in light of (7). First, $\epsilon_{x,Y}$ is less than one. Thus a one percent change in aggregate output results in a less-than-one percent change in relative wage. Second, $\epsilon_{x,Y}$ decreases in both σ and ξ. These observations are the key to understanding the model's potentials in generating persistence.

The above findings are fairly intuitive. Since there is an intertemporal smoothing incentive in labor supply, i.e., $\xi > 0$, a larger σ implies a smaller wage adjustment in response to the shock. This is so because, when it is easier to substitute one type of labor for another, a given relative wage adjustment is associated with a larger employment fluctuation. On the other hand, given that σ is larger than one, a stronger incentive of a household to smooth its labor hours (i.e., a higher ξ) makes it less willing to adjust its relative wage.

We now analyze the N-period wage setting rule, with the intertemporal forward- and backward-looking effects taken into account. The first order condition with respect to the N-period wage decision implies that

$$W(i,s^t) = \frac{\sigma}{\sigma-1}\frac{\sum_{\tau=t}^{t+N-1}\sum_{s^\tau}\beta^{\tau-t}\pi(s^\tau|s^t)(-V_l(i,s^\tau))L^d(i,s^\tau)}{\sum_{\tau=t}^{t+N-1}\sum_{s^\tau}\beta^{\tau-t}\pi(s^\tau|s^t)[U_c(i,s^\tau)/P\,(s^\tau)]L^d(i,s^\tau)},$$

where $\pi(s^\tau|s^t) = \pi(s^\tau)/\pi(s^t)$ is the conditional probability of s^τ given s^t, for $\tau \geq t$. Hence, the household's optimal wage is a constant "markup" over the ratio of weighted marginal utilities of leisure to those of income over the contract periods, where the weights are given by normalized quantities demanded for its labor services. Clearly, this equation reduces to (5) when $N = 1$.

To gain further insights into this wage decision rule, it is helpful to examine the log-linearized version of the wage setting equation

$$w_t = \sum_{j=1}^{N-1} b_j w_{t-j} + \mathrm{E}_t \sum_{j=1}^{N-1} b_j w_{t+j} + \frac{\gamma}{N-1}\mathrm{E}_t \sum_{j=0}^{N-1} y_{t+j}, \tag{8}$$

where the lower-case variables denote log-deviations of the corresponding upper-case variables from their steady state values, E_t is a conditional expectation operator, and the event argument s^t is replaced by the time subscript t to save notation.

We have also set $\beta = 1$ to simplify the expressions. The weights on lagged and forward wages in (8) are given by $b_j = \frac{N-j}{N(N-1)}$, and the coefficient in front of current and future outputs is given by

$$\gamma = \frac{1+\bar{\xi}}{1+\sigma\bar{\xi}}, \tag{9}$$

where $\bar{\xi}$ is the household's steady state relative risk aversion in labor hours. Accordingly, γ is the steady state counterpart of $\epsilon_{x,Y}$.

Equation (8) is apparently identical to Taylor's (1980) structural equation, except that the parameter γ in his model is a structural parameter, while it is here a parameter determined by the underlying preferences and technologies. It is clear from (8) that when a household sets a new wage, it looks at both the wages set in the previous $N-1$ periods and those expected to be set in the future $N-1$ periods. Since b_j is declining in j, the household assigns lower weights to those wages set either in the further past or in the further future. This backward- and forward-looking consideration implies that the household would like to keep in line with the peers when deciding on its own wage, as emphasized by Taylor (1980).

More importantly, a household that can set a new wage takes into account changes in aggregate demand conditions during its contract periods. The parameter γ measures the responsiveness of the household's wage to such changes. A smaller γ implies a slower wage adjustment, and thus more output persistence. Equation (9) shows that γ depends on both the elasticity of substitution among differentiated labor skills and the steady state relative risk aversion in labor hours. Given that $\bar{\xi} > 0$ and $\sigma > 1$, γ is necessarily less than one and decreases with both $\bar{\xi}$ and σ. Thus the staggered wage mechanism can potentially contribute to generating persistence.[6]

To illustrate the role of γ in helping generate persistence, we derive analytical solutions of equilibrium output dynamics in the case with $N = 2$. Equation (8) then simplifies to[7]

$$w_t = \frac{1}{2}w_{t-1} + \frac{1}{2}\mathrm{E}_t w_{t+1} + \gamma(y_t + \mathrm{E}_t y_{t+1}).$$

Combining this equation with the log-linearized money demand equation $p_t + y_t = m_t$, and the zero-profit condition $p_t = \bar{w}_t = (w_t + w_{t-1})/2$, we obtain a second order difference equation in w_t

$$\mathrm{E}_t w_{t+1} - \frac{2(1+\gamma)}{1-\gamma}w_t + w_{t-1} = -\frac{2\gamma}{1-\gamma}\mathrm{E}_t(m_t + m_{t+1}).$$

With an additional assumption that m_t follows a random walk process, the solution of this difference equation is $w_t = aw_{t-1} + (1-a)m_t$. The implied dynamic output equation is given by (3), as in the simplified version of Taylor's (1980) model. The only difference is that the key persistence parameter γ is here determined by preference and technology parameters $\bar{\xi}$ and σ, as described in (9).

4 A Model with Staggered Price Contracts

In this section, we present a general equilibrium model with staggered price contracts. As will be shown, the dynamic price setting equation in this model is apparently identical to the dynamic wage setting equation in the staggered wage model presented above. However, the elasticity of relative price with respect to aggregate output, the counterpart of γ in the previous model, is here linked to the underlying economic fundamentals in a different way, and the model is not able to deliver any persistence for reasonable parameter values.

The model is a simplified version of CKM (1998). To be specific, there is a continuum of firms who use homogeneous labor services to produce differentiated goods indexed in the interval $[0,1]$, and there is a representative household who supplies the labor and consumes a composite of all types of the goods.

The household's utility function is given by

$$U = \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) \log(C^\star(s^t)) + V(L(s^t)),$$

where $C^\star(s^t) = \left[bC(s^t)^\nu + (1-b)(M(s^t)/\bar{P}(s^t))^\nu\right]^{1/\nu}$ is a CES composite of consumption and real money balances, $V(\cdot)$ is strictly decreasing and strictly concave, $\bar{P}(s^t)$ is a price index, and $C(s^t) = \left[\int_0^1 Y(j,s^t)^{\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}} \equiv Y(s^t)$ is a composite of all types of differentiated goods. Here, $Y(j,s^t)$ is the output of firm j at s^t, and θ is the elasticity of substitution among different types of goods, where $\theta > 1$.

Upon the realization of s^t, the household solves the utility maximization problem by choosing consumption goods $\{Y(j,s^t)\}_{j\in[0,1]}$, nominal money balances $M(s^t)$, and one-period nominal bond holdings $B(s^{t+1})$, taking prices $\{P(j,s^t)\}_{j\in[0,1]}$ and $D(s^{t+1}|s^t)$, and nominal wage $W(s^t)$ as given. The utility maximization is subject to a sequence of budget constraints

$$\int_0^1 P(j,s^t)Y(j,s^t)dj + \sum_{s^{t+1}} D(s^{t+1}|s^t)B(s^{t+1}) + M(s^t)$$

$$\leq W(s^t)L(s^t) + \Pi(s^t) + B(s^t) + M(s^{t-1}) + T(s^t),$$

and a borrowing constraint similar to that in Section 3. From the first order conditions we derive the demand function for good j

$$Y^d(j, s^t) = \left(\frac{P(j, s^t)}{\bar{P}(s^t)} \right)^{-\theta} Y(s^t), \tag{10}$$

and the optimal labor supply decision

$$\frac{-V_l(s^t)}{U_c(s^t)} = \frac{W(s^t)}{\bar{P}(s^t)}, \tag{11}$$

where $\bar{P}(s^t) = \left(\int_0^1 P(j, s^t)^{1-\theta} dj \right)^{\frac{1}{1-\theta}}$ is the price index.

Production technology for firm j is given by $Y(j, s^t) = L(j, s^t)$, where $L(j, s^t)$ is the labor used by j at s^t. Firms are price-takers in labor markets and monopolistic competitors in goods markets. They take the goods demand schedule (10) as given and set prices in a staggered fashion. All firms are divided into N cohorts based on the timing of their price setting. Upon the realization of s^t, a firm j that can set a new price solves an N-period profit maximization problem

$$\text{Max}_{P(j,s^t)} \sum_{\tau=t}^{t+N-1} \sum_{s^\tau} D(s^\tau | s^t) \left[P(j, s^t) - W(s^\tau) \right] Y^d(j, s^\tau),$$

subject to (10). The resulting optimal pricing rule is given by

$$P(j, s^t) = \frac{\theta}{\theta - 1} \frac{\sum_{\tau=t}^{t+N-1} \sum_{s^\tau} D(s^\tau | s^t) \bar{P}(s^\tau)^\theta W(s^\tau) Y(s^\tau)}{\sum_{\tau=t}^{t+N-1} \sum_{s^\tau} D(s^\tau | s^t) \bar{P}(s^\tau)^\theta Y(s^\tau)}.$$

Thus the firm's optimal price is a markup over a weighted average of the marginal costs during its contract periods, where the marginal costs are given by the nominal wages since labor is the only input.

Assuming that monetary policy and money demand equation are the same as in the staggered wage model, we can define an equilibrium analogously. In what follows, we focus on a symmetric equilibrium in which firms in the same cohort make identical price decisions. The main finding is that marginal cost always changes more than aggregate output does in response to a shock, and thus a firm always fully adjusts its price whenever it gets the chance to set a new price. In consequence, price level changes quickly, and aggregate output returns to steady state as soon as every firm gets the chance to renew its contract.

To understand this no-persistence result, it is essential to understand how equilibrium real wage, the real marginal cost in this model, responds to changes in aggregate output. For this purpose, we rewrite the labor supply equation (11) as

$$\frac{W(s^t)}{\bar{P}(s^t)} = \left(\frac{1}{b}\right) \left[-V_l(L^s(s^t))\right] Y\left(s^t\right). \tag{12}$$

The labor demand function is given by

$$L^d(s^t) = \int_0^1 L(j,s^t)dj = \int_0^1 Y^d(j,s^t)dj = \left[\int_0^1 \left(\frac{P(j,s^t)}{\bar{P}(s^t)}\right)^{-\theta} dj\right] Y(s^t) \equiv G(s^t)Y(s^t), \tag{13}$$

where the second equality follows from the production function, the third equality follows from the output demand function, and the final equality defines $G(s^t)$. Labor market equilibrium requires that labor supply $L^s(s^t)$ in (12) equal labor demand $L^d(s^t)$ in (13). This equality determines an equilibrium real wage.

Figure 1 illustrates labor market equilibria before and after an aggregate demand shock, where aggregate output Y is a shift variable. In Figure 1, a change in aggregate output from Y_0 to Y_1 leads to a shift in both labor supply and labor demand curves. The labor supply equation (12) reveals that, for a given labor demand, a one percent increase in Y causes an equal percentage increase in real wage (from point **A** to **B** in the diagram). The labor demand equation (13) reveals that an increase in Y causes a one-for-one increase in labor demand, thus shifts the labor demand curve to the right and further pushes up real wage via moving along the new labor supply curve (from point **B** to **C**). By taking total differentiation of (12), we find that the magnitude of this second increase in real wage equals $\xi \equiv V_{ll}L/V_l$, the household's relative risk aversion in labor hours. The total percentage increase in real wage due to a one percent increase in aggregate output (from **A** to **C**) is then given by

$$\epsilon_{w,Y} \equiv \frac{\partial(W/\bar{P})}{\partial Y}\frac{Y}{(W/\bar{P})} = 1 + \xi. \tag{14}$$

Given that the household is risk averse in labor hours, i.e., $\xi > 0$, $\epsilon_{w,Y}$ is necessarily larger than one. Thus, real wage rises by more than aggregate output does. Facing such a large increase in real marginal cost, each firm fully raises its price whenever it gets the chance to set a new price. Price level thus rises quickly and the output response is short-lived.

This inability of the staggered price mechanism is in contrast to the potential of the staggered wage mechanism in generating persistence. Nonetheless, confusion of the two mechanisms may arise from the apparent similarity of the linearized decision rules in the two models. The log-linearized price equation in the current model, by setting $\beta = 1$, is given by

$$p_t = \sum_{j=1}^{N-1} b_j p_{t-j} + \mathrm{E}_t \sum_{j=1}^{N-1} b_j p_{t+j} + \frac{\gamma}{N-1} \mathrm{E}_t \sum_{j=0}^{N-1} y_{t+j},$$

which is apparently identical to the log-linearized wage equation (8) in the staggered wage model, with w_t being replaced by p_t everywhere. Indeed, the coefficients b_j are identical in the two equations so that the intertemporal backward- and forward-looking effects seem to work in the same way under the two mechanisms. However, the parameter γ is determined in different ways across the two models so that the optimal wage and the optimal price responses to changes in aggregate demand conditions are different. In the staggered price model, γ is the steady state counter part of $\epsilon_{w,Y}$ and is given by

$$\gamma = 1 + \bar{\xi}, \tag{15}$$

where $\bar{\xi}$ is households' steady state relative risk aversion in labor hours. Given that $\bar{\xi} > 0$, the parameter γ is here necessarily larger than one and increases with $\bar{\xi}$. Thus, the staggered price model is not capable of generating persistence.

To make this no-persistence result more transparent, we solve for the equilibrium output dynamics when $N = 2$, and the solution is again given by (3). Since γ is here greater than one, the value of a is necessarily negative and there is no persistence.

In light of (9) and (15), as long as there is an intertemporal smoothing incentive in labor supply (i.e., $\bar{\xi} > 0$), the key persistence parameter (γ) in the two models is linked to preferences and technologies in two different ways, rendering the two models different potentials in generating persistence.

5 Models with Intertemporal Links

As our analytical results in Sections 3 and 4 have shown, the staggered price mechanism does not contribute to generating persistence while the staggered wage mechanism potentially can. As shown in CKM (1998), the inability of the staggered price model in generating persistence is robust when there are intertemporal links such as capital accumulation and interest rate sensitive money demand. To assess the quantitative contribution of the staggered wage mechanism to generating persistent real effects of monetary shocks, we examine a calibrated version of the staggered wage model with these intertemporal links. We find that the staggered wage mechanism does play an important role in generating persistence.

We describe the formal model, the computation methods, and the calibration strategies in the Appendix, and present the main results in this section. Since analytical solutions are difficult to obtain, we resort to numerical methods to solve the log-linearized equilibrium conditions. The calibrated parameter values are shown in Table 1. All parameters are calibrated using standard methods as in CKM (1998), except for the elasticity of substitution among differentiated labor skills in the staggered wage model (σ). We set $\sigma = 6$ in light of the micro-studies by Griffin (1992, 1996).[8]

In what follows, we report the impulse response functions of the models' key variables following a monetary shock. The money supply process is given by $M(s^t) = \mu(s^t)M(s^{t-1})$, and the money growth rate follows the process

$$\ln \mu(s^t) = \rho \ln \mu(s^{t-1}) + \varepsilon_t, \qquad (16)$$

where $0 < \rho < 1$, and ε_t has an i.i.d. normal distribution with zero mean and finite variance. To compute the impulse responses, we choose the magnitude of the innovation term in the money growth rate (the ε_t term) so that money stock rises by 1% one year after the shock.

Figure 2 plots the impulse response functions of output in the two models with $N = 4$. In the staggered price model, the output initially rises, and then returns to steady state as soon as the initial contract expires (i.e., one year after the shock). This finding is consistent with CKM (1998). In contrast, the output response in the staggered wage model is much more persistent. To measure the magnitude of persistence, we define a "contract multiplier" as the ratio of the output response at the end of the initial contract duration to that in the impact period. The contract multiplier is negative under the staggered price mechanism whereas it is about 40% under the staggered wage mechanism.

Figures 3 and 4 display the impulse responses of key variables in the two models. In both models, consumption, investment, and employment are all procyclical. Investment is more volatile than output, which in turn is more volatile than consumption. The nominal interest rate and inflation rate are both procyclical. Interestingly, all these are standard features of a monetary business cycle model without nominal rigidities (e.g. Cooley and Hansen (1995)). Except for the lack of "liquidity effect," these features are broadly consistent with the business cycle facts in the U.S. economy. Nonetheless, the two models' equilibrium predictions differ in two key aspects. First, the impulse responses of both real and nominal variables in the staggered wage model are more persistent than those in the staggered price model. Second, real wage is strongly procyclical in the staggered price model, while it is weakly countercyclical in the staggered wage model. Evidence on the cyclicality of real wage is mixed. As surveyed by Abraham and Haltiwanger (1995), existing empirical studies do not suggest systematically procyclical or countercyclical real wages.[9]

Finally, Figure 5 displays the impulse response of output in the staggered wage model with different values of σ and different N. In addition to our benchmark value of $\sigma = 6$, higher values of this parameter are reported in the literature. For example, Erceg (1997) uses a value of 10, Kim (1996) obtains an estimate of 12, and Koenig (1997) argues that σ can be as high as 20. As shown in Figure 5, a larger σ leads to flatter output responses and hence more persistence. Additionally, a larger N also produces more persistence. It is interesting to note that, the staggered wage model is able to generate a hump-shaped output response for σ values within the range reported in the literature. The findings here confirm the analytical results in Section 3, and are consistent with Taylor's (1980) original insights that a larger degree of asynchronization in wage setting (i.e., a larger N) generates more output persistence.

To summarize, the basic insights elaborated by our analytical solutions in Sections 3 and 4 stand up to the incorporation of such intertemporal links as capital accumulation and interest rate sensitive money demand. While the staggered price mechanism by itself does not contribute to, the staggered wage mechanism plays an important role in generating persistence.

6 Conclusion

We have shown that, with optimizing individuals, staggered wage contracts and staggered price contracts have different implications on persistence. Although the dynamic price setting and the dynamic wage setting equations in the two models are apparently identical, the key parameter that governs persistence in the two equations is linked to preferences and technologies in different ways, resulting in different predictions on how aggregate output responds to monetary shocks. While the staggered price model *by itself* does not contribute to, the staggered wage model plays an important role in generating persistence. The difference between the two mechanisms cannot possibly be uncovered unless individuals' optimizing behaviors are explicitly modeled.

We have focused on distinguishing the two mechanisms in their abilities of generating persistence, and have not attempted to propose a single friction model that is able to fully account for the dynamic output responses to monetary shocks. As suggested by Christiano, et. al (1997), it is unlikely for a single-friction model to provide a complete account of the real effects of monetary shocks. To provide such an account, a combination of frictions is required. Our findings in this paper suggest that, in such a multi-friction model, staggered wage contracts can be an important contributing mechanism.

Appendix

This appendix presents a model of staggered wage contracts with capital accumulation. The model is identical to the model in Section 3 with two exceptions. First, firms' production requires both labor and capital as inputs. Second, households' problems now involve decisions on capital accumulation. The model of staggered price contracts with capital accumulation is not formally presented here because it is similar to CKM (1998).

A.1. The Model

We begin with firms' problems. Each firm has access to a Cobb-Douglas production function

$$F(K(s^t), L(s^t)) = K(s^t)^{\alpha} L(s^t)^{1-\alpha}, \tag{17}$$

where $0 < \alpha < 1$, $K(s^t)$ is the capital stock at s^t, and $L(s^t) = \left[\int_0^1 L(i, s^t)^{\frac{\sigma-1}{\sigma}} di\right]^{\frac{\sigma}{\sigma-1}}$ is a composite of labor services. Let $R^k(s^t)$ denote the nominal rental rate on capital. By minimizing the production cost $R^k(s^t)K + \int_0^1 W(i, s^t)L(i)di$ subject to (17), we obtain the demand functions for $L(s^t)$, $K(s^t)$, and $L(i, s^t)$. The resulting marginal cost function is $MC(s^t) = \tilde{\alpha}\bar{W}(s^t)^{1-\alpha}R^k(s^t)^{\alpha}$, where $\tilde{\alpha} = \alpha^{-\alpha}(1-\alpha)^{\alpha-1}$. Profit maximization implies that price equals marginal cost, that is,

$$P(s^t) = \tilde{\alpha}\bar{W}(s^t)^{1-\alpha}R^k(s^t)^{\alpha}. \tag{18}$$

We next specify households' problems. The utility function is the same as in the baseline model. The budget constraint is now given by

$$\begin{aligned} & P(s^t)C(i, s^t) + P(s^t)I(i, s^t)\left[1 + \phi\left(\frac{I(i, s^t)}{K(i, s^{t-1})}\right)\right] + \sum_{s^{t+1}} D(s^{t+1}|s^t)B(i, s^{t+1}) + M(i, s^t) \\ \leq\ & W(i, s^t)L^d(i, s^t) + R^k(s^t)K(i, s^{t-1}) + \Pi(i, s^t) + B(i, s^t) + M(i, s^{t-1}) + T(i, s^t), \end{aligned} \tag{19}$$

where $I(i, s^t)$ and $\phi(I(i, s^t)/K(i, s^{t-1}))$ are the investment and the capital adjustment cost of household i in s^t, respectively. Capital accumulation is governed by

$$I(i, s^t) = K(i, s^t) - (1-\delta)K(i, s^{t-1}), \tag{20}$$

where $\delta \in (0,1)$ is a capital depreciation rate.

Household i maximizes utility choosing $C(i,s^t)$, $I(i,s^t)$, $M(i,s^t)$, and $B(i,s^{t+1})$, subject to (19)-(20) and a borrowing constraint $B(i,s^t) \geq -\bar{B}$ for some large positive number $\bar{B}$, taking prices $P(s^t)$, $\bar{W}(s^t)$, $R^k(s^t)$, and $D(s^{t+1}|s^t)$ and initial conditions $K(i,s^{-1})$, $M(i,s^{-1})$, and $B(i,s^0)$ as given. If the household is a member of the cohort that can set new wages, it also chooses a nominal wage $W(i,s^t)$ for its contract periods. To simplify notation, we denote by $Q(i,s^t)$ the investment-capital ratio $I(i,s^t)/K(i,s^{t-1})$ and by $H(Q)$ the effective cost of capital $1+\phi(Q)+Q\phi'(Q)$. The first order conditions are

$$U_c(i,s^t) = \lambda(i,s^t)P(s^t), \tag{21}$$

$$U_m(i,s^t)/P(s^t) = \lambda(i,s^t) - \beta\sum_{s^{t+1}}\pi(s^{t+1}|s^t)\lambda(i,s^{t+1}), \tag{22}$$

$$D(s^{t+1}|s^t) = \beta\pi(s^{t+1}|s^t)\lambda(i,s^{t+1})/\lambda(i,s^t), \tag{23}$$

$$\begin{aligned} U_c(i,s^t)H(Q(i,s^t)) = {} & \beta\sum_{s^{t+1}}\pi(s^{t+1}|s^t)U_c(i,s^{t+1})\{R^k(s^{t+1})/P(s^{t+1}) + \\ & (1-\delta)H(Q(i,s^{t+1})) + Q(i,s^{t+1})^2\phi'(Q(i,s^{t+1}))\}, \end{aligned} \tag{24}$$

$$\begin{aligned} & \sum_{\tau=t}^{t+N-1}\sum_{s^\tau}\beta^{\tau-t}\pi(s^\tau|s^t)[-V_l(i,s^\tau)]\frac{\partial L^d(i,s^\tau)}{\partial W(i,s^t)} \\ & = \sum_{\tau=t}^{t+N-1}\sum_{s^\tau}\beta^{\tau-t}\pi(s^\tau|s^t)\lambda(i,s^\tau)L^d(i,s^\tau)(1-\sigma), \end{aligned} \tag{25}$$

where $U_c(i,s^t)$, $U_m(i,s^t)$, and $-V_l(i,s^t)$ denote the marginal utility of consumption, real money balances, and leisure, respectively, $\lambda(i,s^t)$ is the Lagrangian multiplier associated with the budget constraint, and $\pi(s^\tau|s^t) = \pi(s^\tau)/\pi(s^t)$ is the conditional probability of s^τ given s^t, for $\tau \geq t$.

Equations (21)-(24) are standard first order conditions with respect to the household's choice of consumption, money balances, bond holdings, and capital investment, respectively. Equation (25) corresponds to the wage setting rule. The left-hand side of this equation is the expected present value of marginal utility gains due to an increase in wage and thus reduced labor hours during the contract periods, while the right-hand side is the expected present value of marginal utility losses due to unemployed hours and thus a lower wage income. The wage is set to balance the gains and the losses at the margin. Since there are complete contingent asset markets, each household's consumption and money balance decisions depend only on initial distributions of wealth. Without loss of generality, we assume that the initial holdings of wealth are identical across households. This assumption, along with the assumption that consumption and leisure are additively separable in the utility function, implies that the equilibrium consumption and money balances are identical across households for each realization of s^t. That is, $C(i, s^t) = C(s^t)$ and $M(i, s^t) = M(s^t)$. In consequence, $\lambda(i, s^t) = \lambda(s^t)$ for all i, and thus the wage decision rule implied by (25) depends only on aggregate variables.

Capital market clearing requires that $\int_0^1 K(i, s^{t-1})\, di = K(s^t)$, and goods market clearing implies that

$$C(s^t) + I(s^t)\left[1 + \phi\left(\frac{I(s^t)}{K(s^{t-1})}\right)\right] = K(s^t)^{\alpha} L(s^t)^{1-\alpha}. \tag{26}$$

Note that, in each period t, firms' decisions on capital demand are made after the realization of s^t, while the capital stock available for rent is chosen by households at s^{t-1}.

The rest of the optimization conditions is the same as in Section 3. Given the money supply process (16), an equilibrium can be defined analogously.

A.2. The Computation

We now describe how to compute equilibrium decision rules. With appropriate substitutions, the equilibrium conditions can be reduced to three equations, including a wage setting equation, a capital Euler equation, and a money demand equation. The decision variables are current wages, aggregate consumption, and aggregate capital stock. We focus on a symmetric equilibrium in which households in the same cohort make identical decisions. In a symmetric equilibrium, a household's wage decision depends only on the time at which it can set a new wage but not on the index of its labor service. Thus, we have $W(i, s^t) = W(s^t)$ for all i and the wage index is given by

$$\bar{W}(s^t) = \left[\frac{1}{N}W(s^{t-N+1})^{1-\sigma} + \frac{1}{N}W(s^{t-N+2})^{1-\sigma} + \cdots + \frac{1}{N}W(s^t)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \tag{27}$$

We now rewrite (25) as an equation in the three decision variables. To begin, we first use (4) to express $L^d(i, s^\tau)$ and $\partial L^d(i, s^\tau)/\partial W(i, s^t)$ by $W(s^t)$, $\bar{W}(s^\tau)$, and $L(s^\tau)$. We then use (21) to replace $\lambda(i, s^\tau)$ by $C(s^\tau)$, $M(s^\tau)$, and $P(s^\tau)$. Finally, we use (18), (20), (26), and (27) to express $P(s^\tau)$, $L(s^\tau)$, and $\bar{W}(s^\tau)$ by $W(s^\tau)$, $C(s^\tau)$, and $K(s^\tau)$, for $\tau = t, t+1, \ldots, t+N-1$. We also use (18) and the relation $R^k(s^t) = (\alpha/(1-\alpha))(L(s^t)/K(s^{t-1}))\bar{W}(s^t)$ (derived from firms' cost-minimization) to substitute for $P(s^t)$ and $R^k(s^t)$ in (22) and (24), respectively.

Given the Markov money supply process (16), a stationary equilibrium in this economy consists of stationary decision rules which are functions of the state of the economy. In each period t, there are $N-1$ prevailing wages that were set in period $t-N+1$ through period $t-1$ due to staggered wage contracts. Thus, the state of the economy in period t must record the wages set in the previous $N-1$ periods in addition to the beginning-of-period capital stock and the exogenous money growth rate. To induce stationarity, we divide all wages by the money stock. Thus, the state at s^t is given by $[W(s^{t-N+1})/M(s^t), \cdots, W(s^{t-1})/M(s^t), K(s^{t-1}), \mu(s^t)]$.

A.3. The Calibration

In both models, the capital adjustment cost function is given by $\phi(I/K) = (\psi/2)(I/K)^2$ and the utility function takes the form $U(C, M/P, L) = \log[bC^\nu + (1-b)(M/P)^\nu]^{1/\nu} + \eta\log(1-L)$. The parameters to be calibrated include the subjective discount factor β, the preference parameters b, ν, and η, the capital share α, the depreciation rate δ, the adjustment cost parameter ψ, the monetary policy parameter ρ, and the technology parameter (i.e., σ in the staggered wage model and θ in the staggered price model). The calibrated values are summarized in Table 1.

In our baseline model, we set $N = 4$ so that a period in the model corresponds to a quarter. Following the standard business cycle literature, we choose $\beta = 0.96^{1/4}$. To assign values for b and ν, we use the implied money demand equation

$$\log\left(\frac{M(s^t)}{P(s^t)}\right) = -\frac{1}{1-\nu}\log\left(\frac{b}{1-b}\right) + \log(C(s^t)) - \frac{1}{1-\nu}\log\left(\frac{R(s^t)-1}{R(s^t)}\right),$$

where $R(s^t) = \left(\sum_{s^{t+1}} D(s^{t+1}|s^t)\right)^{-1}$ is the gross nominal interest rate. The regression of this equation as performed in CKM (1998) implies that $\nu = -1.56$ and $b = 0.98$ for quarterly U.S. data with a sample range from quarter one in 1960 to quarter four in 1995. The serial correlation parameter ρ of money growth rate is set to 0.57, based on quarterly U.S. data on M1 from quarter three in 1959 to quarter two in 1995 (see also CKM (1998)).

We next choose $\alpha = 0.33$ and $\delta = 1 - 0.92^{1/4}$ so that the baseline model predicts an annualized capital-output ratio of 2.6 and an investment-output ratio of 0.21. The parameter η is selected to match an average share of time allocated to market activity of $1/3$, as in most business cycle studies. We adjust ψ so that the model predicts a standard deviation of aggregate investment to be 3.23 times as large as that of output, in accordance with the U.S. data. Following CKM (1998), we set $\theta = 10$ in the staggered price model, corresponding to a steady state markup of 11%.

Finally, we set $\sigma = 6$ in the staggered wage model, based on the empirical studies by Griffin (1992, 1996), who uses disaggregated firm-level data to estimate the elasticity of substitution among differentiated labor skills.

In the sensitivity analysis conducted in Section 5, we vary the degree of asynchronization in wage setting, N, as well as the labor substitutability parameter, σ. We adjust β, η, b, δ, ψ, and ρ accordingly so as to keep unchanged the labor-leisure ratio, the capital-output ratio, the investment-output ratio, the relative volatility of investment, and the quarterly serial correlation of money growth rate. In particular, we set $\beta = 0.96^{1/N}$, $\delta = 1 - 0.92^{1/N}$, and $\rho = 0.57^{4/N}$, and adjust b, η, and ψ whenever we vary N or σ.

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NOTES

1. Although models with information lags and price stickiness are shown to be quite successful in generating output fluctuations driven by monetary shocks, the resulting effects are usually contemporaneous rather than persistent. See, for example, Lucas (1972), Lucas and Woodford (1993), Rotemberg (1996), and Yun (1996).

2. This view has recently been emphasized by Taylor (1999), who states that "the equations are essentially the same for wage setting and price setting."

3. Our purpose here is to compare the abilities of two alternative mechanisms with staggered nominal contracts in the spirit of Taylor (1980) and CKM (1998) in generating long-lasting endogenous persistence with a short duration of exogenous stickiness. We thus follow these authors and assume time-dependent wage setting and price setting rules.

4. We assume, without loss of generality, that the initial distribution of wealth is identical across all households.

5. Since the labor demand elasticity σ is greater than one, a lower wage $W(i, s^t)$ is associated with higher labor income.

6. The wage decision rule (8) also reveals that the effect of γ on persistence can be reinforced by the number of cohorts. A larger N tends to dampen wage response to changes in current and future aggregate outputs.

7. Notice the similarity of this equation to the price setting rule in Taylor's (1980) simple model described by (1) and (2).

8. The estimate of σ in Griffin (1992, 1996) is based on firm level data representing different industries. As noted by Griffin (1992), the estimate tends to be biased downward for two reasons: (i) all firms in the data set are subject to Affirmative Action which restricts labor substitutability, and (ii) the employment data does not include employee characteristics such as workers' age, experience, and education. Griffin (1996) shows that, when Affirmative Action is explicitly accounted for, the estimate of σ is about 6.

9. Since monetary shocks are the only driving force of fluctuations in our models, to evaluate the models' empirical relevance, we need to compare the models' predictions on real wage behavior with the response of real wage to *monetary* shocks in the data. The evidence is mixed. Some empirical

studies find that real wage is acyclical or weakly procyclical in response to monetary shocks (e.g. Christiano, et al. (1999)), while some other studies suggest the opposite. For example, Bernanke and Carey (1996) find that, using data for 22 countries during the Great Depression, nominal wages adjusted quite slowly to falling prices, resulting in rising real wages amid the dramatic reduction in employment and output. As documented by Friedman and Schwartz (1963), monetary shocks played an important role during the Great Depression. In a survey on the cyclicality of real wages, Abraham and Haltiwanger (1995) note that the real wage cyclicality depends on various factors including the choice of sample periods. For instance, they find that there were roughly synchronized declines in the growth rate of industrial production and real wages in the early to middle 1970s, but in the 1981-82 period, industrial production fell while the real wage growth rate actually increased. It is well known that there was a major monetary contraction during the early 1980s. Our model of staggered wage contracts is more flexible than it appears to be in accommodating the real wage cyclicality. For example, one way to induce acyclical real wages is to add price staggering on top of wage staggering, as in Erceg (1997). But as we have shown elsewhere (Huang and Liu (1999)), adding price staggering in the staggered wage model does not help magnify the persistence.

Table 1.
Calibrated Parameter Values

Preferences:	$\nu = -1.56$
$U(C, M/\bar{P}, L) = \log\left[bC^{\nu} + (1-b)(M/\bar{P})^{\nu}\right]^{1/\nu} + \eta \log(1-L)$	$\eta = 1.41$
	$b = 0.98$
Technologies: $Y = K^{\alpha} L^{1-\alpha}$	$\alpha = 0.33$
Staggered wage model: $L = \left[\int L(i)^{\frac{\sigma-1}{\sigma}} di\right]^{\frac{\sigma}{\sigma-1}}$	$\sigma = 6$
Staggered price model: $Y = \left[\int Y(j)^{\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}}$	$\theta = 10$
Capital Accumulation:	$\delta = 1 - 0.92^{1/4}$
$K_t = I_t + (1-\delta)K_{t-1}$, $\phi(I_t/K_{t-1}) = \psi(I_t/K_{t-1})^2/2$	ψ adjusted
Money Growth: $\log \mu(s^t) = \rho \log(\mu(s^{t-1})) + \varepsilon_t$	$\rho = 0.57$
Subjective discount factor	$\beta = 0.96^{1/4}$
Frequency of Price or Wage Adjustment	$N = 4$

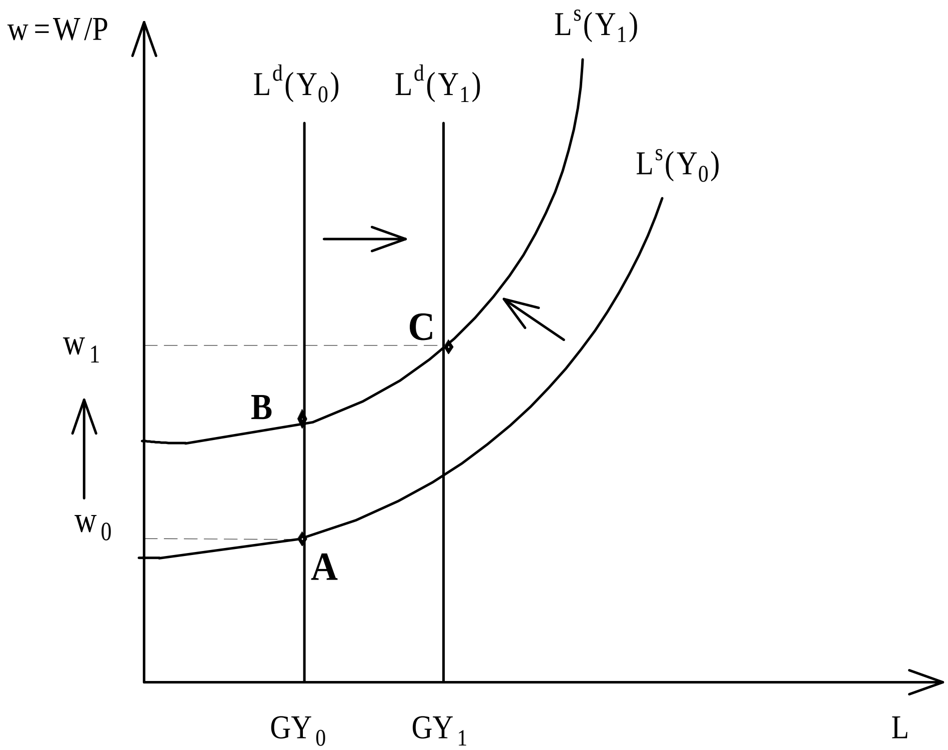

Figure 1:—Real wage response to an aggregate demand shock

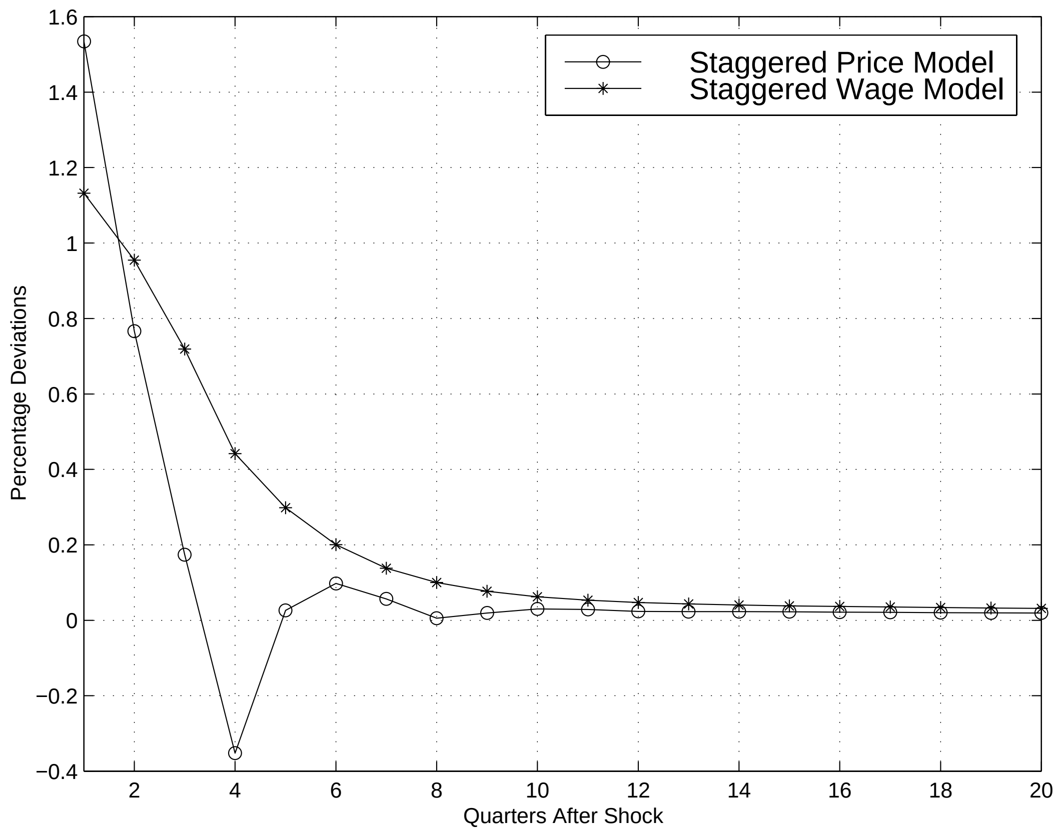

Figure 2:—Impulse response of output in the calibrated models

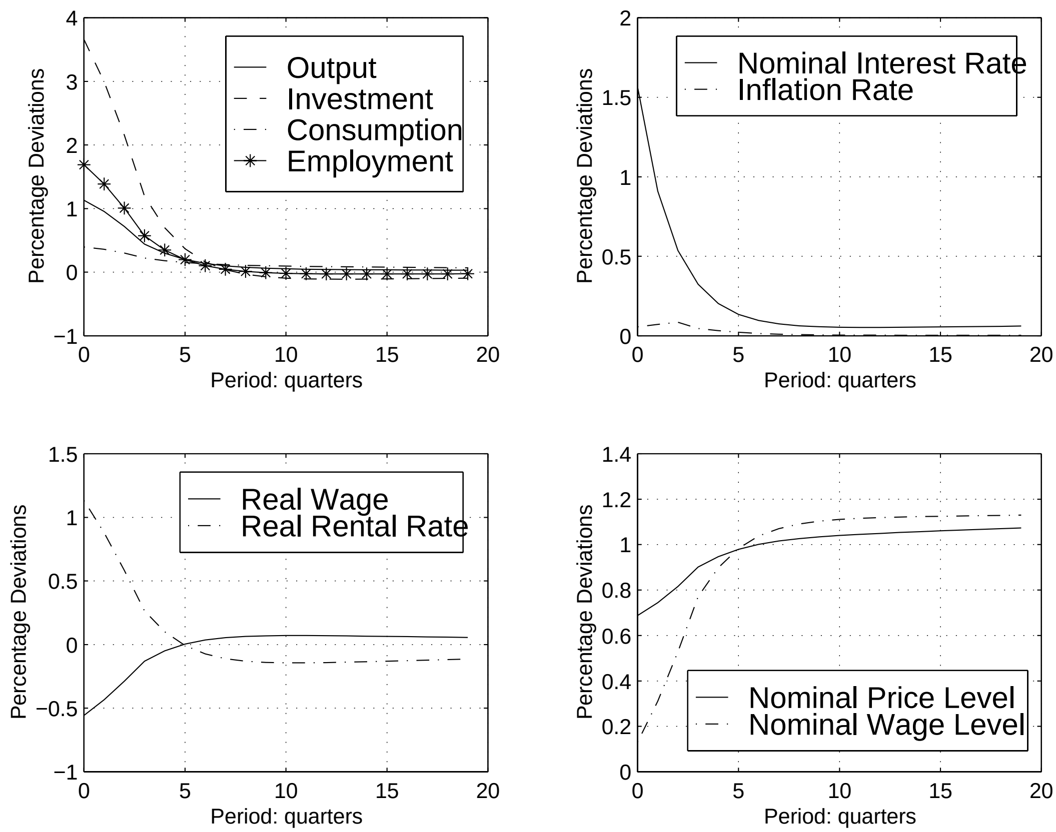

Figure 3:—Impulse responses in the staggered wage model with calibrated parameters

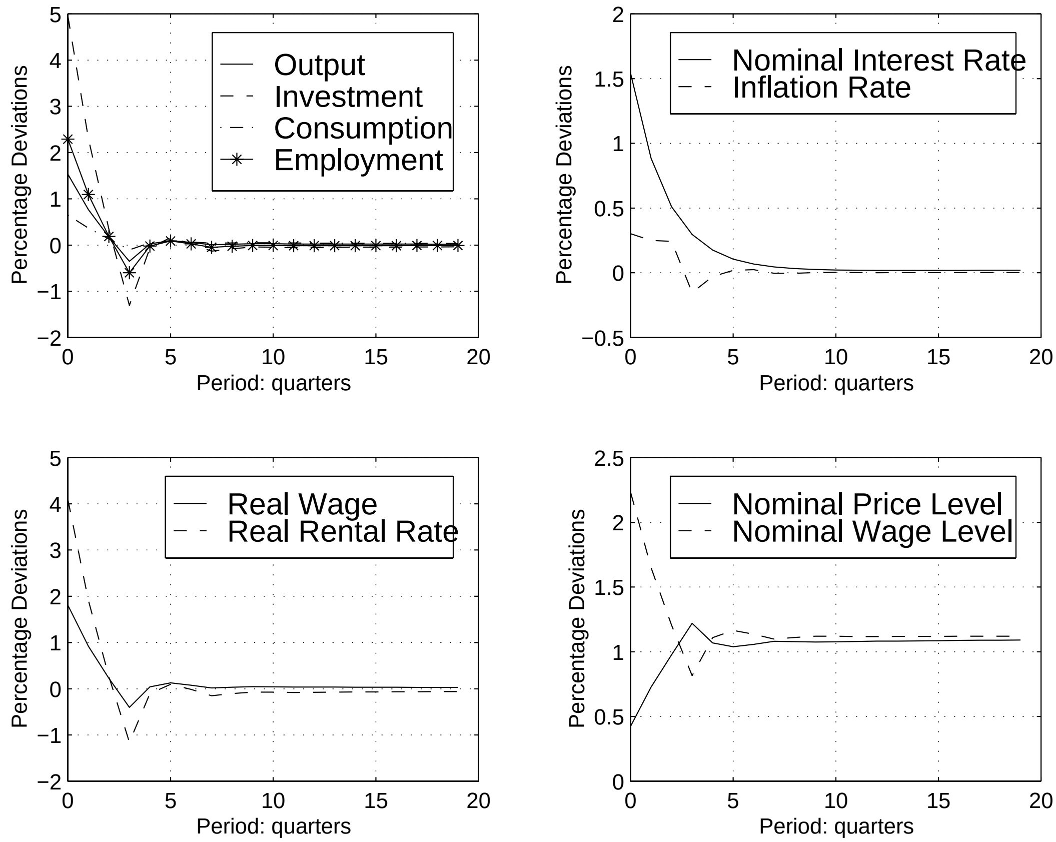

Figure 4:—Impulse responses in the staggered price model with calibrated parameters

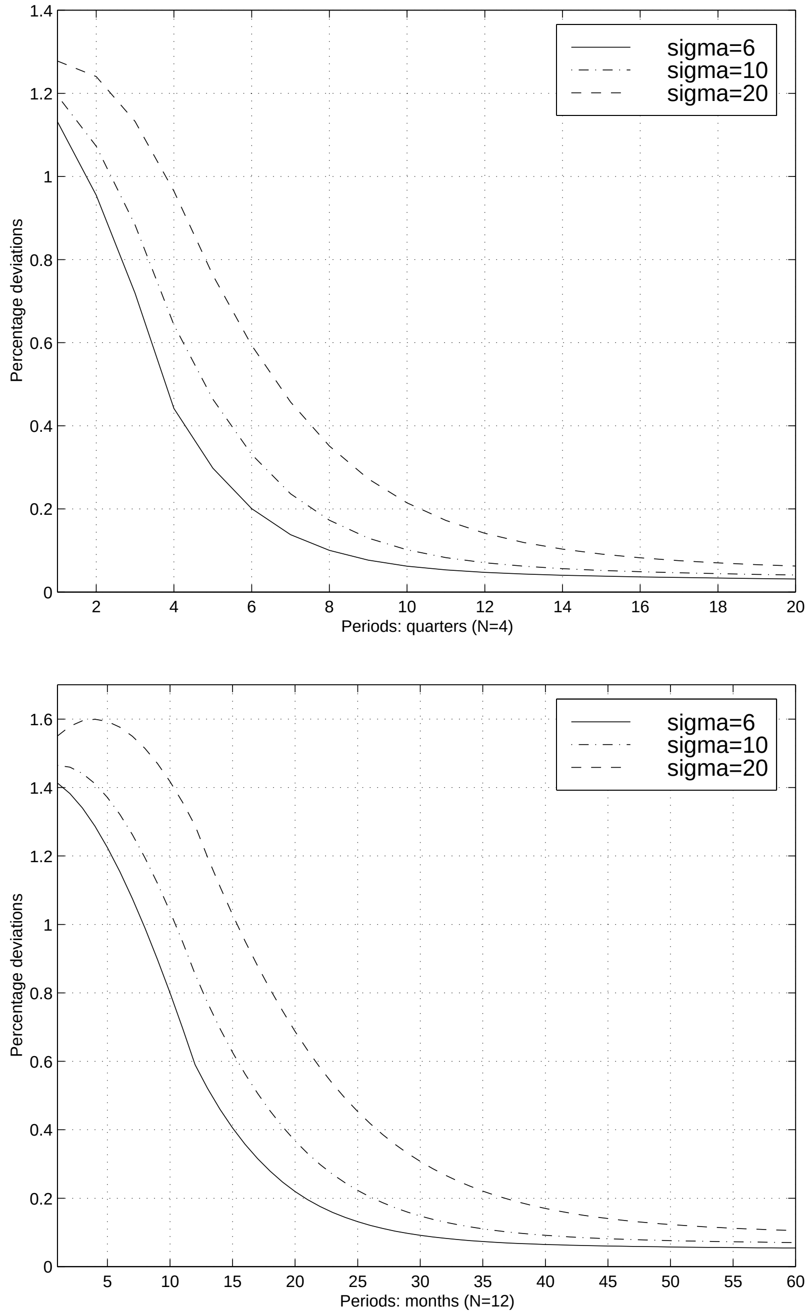

Figure 5:—Impulse responses of output in the staggered wage model with different σ values[a]

[a] The literature provides a wide range of σ values. In addition to Griffin's (1992, 1996) reported σ value of about 6, which is the benchmark value we use, other values are used in the literature. For example, Erceg (1997) uses a value of 10, Kim (1996) obtains an estimate of 12, and Koenig (1997) argues that σ can be as high as 20. The figures here display the impulse response of output for alternative σ values within this range.